Three Essays on Asset Liquidity and its Applications

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The Faculty of Economics, Business Administration and Information Technology of the University of Zurich hereby authorises the printing of this Doctoral Thesis, without thereby giving any opinion on the views contained therein.

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Chairman of the Doctoral Committee: Prof. Dr. Dieter Pfaff

To my wife Pei Yan and my son Tingyi Joe Wang

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Introduction

Liquidity is a complex concept. Stated simply, liquidity is the ease of trading a security. One source of illiquidity is exogenous transaction costs such as brokerage fees, order-processing costs, or transaction taxes. Every time a security is traded, the buyer and/or seller incurs a transaction cost; in addition, the buyer anticipates further costs upon a future sale, and so on, throughout the life of the security.

Another source of illiquidity is demand pressure and inventory risk. Demand pressure arises because not all investors are present in the market at all times, which means that if an investor needs to sell a security quickly, then the natural buyers may not be immediately available. As a consequence, the seller may sell to a market maker who buys in anticipation of being able to later lay off the position. The market maker, being exposed to the risk of price changes while he holds the asset in inventory, must be compensated for this risk - a compensation that imposes a cost on the seller.

Also, trading a security may be costly because the traders on the other side may have private information. For example, the buyer of a stock may worry that a potential seller has private information that the company is losing money, and the seller may be afraid that the buyer has private information that the company is about to take off. Then, trading with an informed counterparty will end up with a loss. In addition to private information about the fundamentals of the security, investors can also have private information about order flow. For instance, if a trading desk knows that a hedge fund needs to liquidate a large position and that this liquidation will depress prices, then the trading desk can sell early at relatively high prices and buy back later at lower prices.

These costs of illiquidity should affect securities prices if investors require compensation for bearing them. In addition, because liquidity varies over time, risk-averse investors may require a compensation for being exposed to liquidity risk. These effects of liquidity on asset prices are important. Investors need to know them in designing their investment strategies. And if liquidity costs and risks affect the required return by investors, they affect corporations cost of capital and, hence, the allocation of the economys real resources.

Liquidity has wide ranging effects on financial markets. It has been previously shown in the literature that liquidity can explain the cross-section of assets with different liquidity, after controlling for other assets characteristics such as risk, and the time series relationship between liquidity and securities returns. Liquidity helps explain why certain hard-to-trade securities are relatively cheap, the pricing of stocks and corporate bonds, the return on hedge funds, and the valuation of closed-end funds. It follows that liquidity can help explain a number of puzzles, such as why equities commanding high required returns (the equity premium puzzle), why liquid risk-free treasuries have low required returns (the risk-free rate puzzle), and why small stocks that are typically illiquid earn high returns (the small firm effect).

This thesis consists of three chapters. Chapter 1 deals with the effect of liquidity risk on firm's capital structure and bond yield spreads, Chapter 2 studies how liquidity risk affects the assessment of the performance of hedge funds, and the object of Chapter 3 is to investigate the correlations between investors' heterogeneous beliefs and the trading volume, price volatility and liquidity of stocks.

In particular, in Chapter 1 we extend the structural model of Leland and Toft (1996) by incorporating liquidity risk. It is expected that introducing liquidity risk into the structural model can generate optimal capital structure and bond yield spreads which are more consistent with empirical findings in the financial markets. We model liquidity risk by assuming that liquidity shocks follow a Poisson process. When bondholders face a liquidity shock, they have to immediately sell the bonds they hold because of borrowing constraints. Transaction costs are assumed to be proportional to bond price, and the proportion is either constant or time-varying. Our results show that incorporating liquidity risk into the structural model can help generate optimal capital structure and yield spreads which are more in line with empirical findings. For some reasonable parameter inputs, the issuance maturities of corporate bonds are 7-8 years, and the optimal leverage ratio can be as low as around 23%. These numbers are closer to the empirical values than those obtained in Leland and Toft (1996). In the meanwhile, the yield spreads of both investment-grade and $'junk'$ bonds increase. In particular, the yield spreads of investment-grade bonds of short maturities are not negligible anymore when one accounts for the effect of liquidity risk.

Chapter 2 presents a joint work with Rajna Gibson (University of Geneva) on the assessment of the performance of hedge funds. Hedge funds have been becoming an important investment vehicle in the financial markets since the 1990s. The crucial issue that investors should take into account in investment decisions is whether hedge funds can deliver significant excess return over the benchmark? In other words, do hedge funds possess any special skills in market timing and asset selection? There are not identical answers to this question. Recently, leveraging on the Bayesian framework proposed by Avramov and Wermers (2006), Avramov et al. (2007) have studied the performance of optimal portfolio strategies in hedge funds. They find that there exist subgroups of hedge funds who possess higher managerial skills, and incorporating predictability in managerial skills can help investors select such hedge funds. We extend this study by examining the joint impact of predictability and of an important omitted risk factor, namely liquidity risk, on the assessment of the performance of hedge funds. Liquidity risk factor is constructed using the liquidity measure developed by Pastor and Stambaugh (2003). We use the Hasanhodzic and Lo (2007) six-factor model to evaluate the ex-post out-of-sample performance of a large number of hedge fund portfolio strategies some of which account for predictability in managerial skills. The main out-of-sample results suggest that once we account for the effect of the omitted liquidity risk factor, the significance of alphas generated by a large number of hedge fund portfolios disappears or is vastly reduced in more than half of TASS hedge fund styles even when predictability in managerial skills is considered. Our empirical results are robust to: (i) the choice of an alternative performance evaluation model (The Fung and Hsieh (2004) seven-factor performance evaluation model), (ii) the choice of an alternative liquidity risk proxy derived from Amihud (2002) liquidity measure, (iii) the exclusion of the January effect, and (iv) the exclusion of the recent financial crises impact. These results indicate that introducing predictability in managerial skills is not sufficient to generate a "pure" and economically significant alpha within most hedge funds investment styles and that liquidity risk plays an important role within many hedge funds styles while a large fraction of the hedge funds superior performance documented by previous performance models actually represents a mere compensation for their liquidity risk exposures.

In Chapter 3, the effects of investors' heterogeneous beliefs on the trading volume, price volatility and liquidity of stocks are investigated. Stock liquidity is one of the most important topics in the Market Microstructure literature. Previous studies suggest that inventory risk and information asymmetry are two key factors which mostly influence stock liquidity: stock liquidity decreases with the inventory risk of dealers and/or market makers and information asymmetry. Following Kurz and Motolese (2008), we propose a model to show that, in addition to inventory risk and information asymmetry, investors' beliefs also have a significant impact on stock liquidity. In particular, we show that stock liquidity is negatively correlated with the volatility of market belief, with liquidity being defined as price pressure and market belief as the average of investors' individual beliefs. Meanwhile, the volatility of market belief also affect the trading volume and price volatility of stocks: higher volatility of market belief is followed by lower trading volume and higher price volatility. We empirically examine these theoretically predicted relations with the analyst forecast data on quarterly earnings per share (EPS) provided by the Institutional Brokers' Estimate System (I/B/E/S). Empirical results well confirm our theoretical findings although the empirical correlation between the volatility of market belief and trading volume is not that significant as predicted in the theoretical model. Our results are robust to diverse methods of estimating market belief and its volatility and to alternative liquidity measure.

Contents

Chapter 1

Optimal Capital Structure and Yield Spreads under Liquidity Risk

1.1 Introduction

Credit risk has been intensively explored in modern finance during the past several decades. One of the most widely employed frameworks of credit risk is the structural approach pioneered by Black and Scholes (1973) and Merton (1994), and subsequently extended by many researchers. The main theoretical structural models include Anderson and Sundaresan (1996), Black and Cox (1976), Collin-Dufresne and Goldstein (2001), Duffie and Lando (2001), Leland (1994), Leland and Toft (1996), Longstaff and Schwartz (1995), Mella-Barral and Perraudin (1997) and Zhou (2001). However, a widespread view amongst financial economists is that the structural models of credit risk, although theoretically appealing, generate yield spreads lower than the actual ones.

The reduced-form approach, developed by Duffie and Singleton (1999), Jarrow et al. (1997) and Madan and Unal (1994), treats default as an unpredictable Poisson event and generates the rich dynamics of the term structure of yield spreads. It is not clear from this approach what economic mechanism is behind the default process and therefore few theoretical insights on the causes of the term structure dynamics of yield spreads can be provided.

A number of papers have studied the determinants of corporate yield spreads and found that credit risk alone could not explain total yield spreads observed. Among them, Jones et al. (1984) is probably the first paper which shows that the large yield spreads of corporate bonds could not be explained by the default risk. Huang and Huang (2003) shows that credit risk accounts for only a small fraction of the observed yield spreads of investment-grade corporate bonds of all maturities - typically around 20%, with an even smaller fraction for bonds of shorter maturities, and that it accounts for a much higher fraction of yield spreads for $'junk'$ bonds. Longstaff et al. (2004) shows that credit risk component represents the majority of corporate yield spreads, however, they also find evidence of a significant non-default component in corporate bonds in the meanwhile. More evidence can be found in Yu (2000) and Ericsson et al. (2006).

The other problem which is important but less intensively discussed in the structural models is the optimal capital structure of a firm. The optimal capital structure implied by the structural models is not consistent with what is observed in financial markets. Leland and Toft (1996) predict that the optimal leverage ratio of a firm increases with the maturity of corporate bonds and can be as high as 50%, which is much higher than the actual one. In addition, their results also show that firms prefer to issue bonds of long maturities. In reality, the average maturity of corporate bonds has been declining since the 1990's and was around 7 years in 2005[1]. Although Goldstein, Ju and Leland (2001) reduces the size of leverage ratio by considering an EBIT-based model of a dynamic capital structure, it still remains too high.

In this paper, we extend the structural model of Leland and Toft (1996) by incorporating liquidity risk. We believe that integrating liquidity risk into the structural model can yield capital structure and yield spreads which are more consistent with empirical findings.

Indeed, liquidity risk has long been perceived as one of the main justifications for the existence of yield spreads above benchmark Treasury notes or bonds since Fischer (1959). Recently, Longstaff et al. (2004) and Ericsson et al. (2006) both find that the non-default component of corporate yield spreads is closely related to the liquidity of corporate bonds. Furthermore, as explained by Longstaff et al. (2004), liquidity risk is important since it may also help explain why firms tend to issue less debt in their capital structure than models that only consider the trade-off between the costs of financial distress and the tax benefits of debt. Despite of its importance, liquidity risk remains a relatively unexplored topic, in particular, liquidity risk for defaultable securities[2].

We model liquidity risk by assuming that bondholders may encounter liquidity shocks during the lifetime of the bonds and that the probability of liquidity shocks follows a Poisson distribution. When facing a liquidity shock, bond holders are assumed to immediately sell the bonds they hold at a discounted price. Liquidity costs are first assumed to be constant and then to follow a mean-reverting diffusion process.

Our results show that incorporating liquidity risk into the structural model can help generate optimal capital structure and yield spreads which are more in line with empirical findings. For some reasonable parameter inputs, the issuance maturities of corporate bonds are greatly shortened to 7-8 years and the optimal leverage ratio is reduced to as low as around 23%. These numbers are closer to the empirical ones mentioned above. In the meanwhile, the yield spreads of both investment-grade and $'junk'$ bonds increase. In particular, the yield spreads of investment-grade bonds of short maturities are not negligible anymore when one considers liquidity risk.

The remainder of this paper is structured as follows: a brief review of literature on the effects of liquidity and liquidity risk on asset returns is given in Section 2; section 3 first repeats the basic results of Leland and Toft (1996) and then extends this model to

[1] Here, corporate bond includes all non-convertible corporate debt, MTNs and Yankee bonds, but excludes all issues with maturities of one year or less, CDs and federal and agency debt. The statistics can be found in the website of www.bondmarkets.com.

[2] In Section 2, we summarize some recent empirical and theoretical papers about the effects of liquidity risk on the prices and yield spreads of securities.

incorporate liquidity risk; the numerical results can be found in Section 4; Section 5 is the conclusion.

1.2 Review of Literature

Liquidity is a complex concept. Stated simply, liquidity is the ease of trading a security. There exist three main sources of illiquidity: One is exogenous transaction costs such as brokerage fees, order-processing costs, or transaction taxes; another is demand pressure and inventory risk, which is due to the temporary imbalance between intended buying and selling; the third is private information, some investors having more information than others. Liquidity is usually associated with the fact that the holding horizon is known in advance.

In a seminal paper, Amihud and Mendelson (1986) examine the effect of liquidity on asset pricing. With the bid-ask spread as a liquidity measure, they analyze a model in which investors with different expected holding periods trade assets with different relative spreads and find that asset returns to their holders, net of trading costs, increase with the spread. Importantly, the liquidity effect is significant. However, in the meanwhile, Constantinides (1986) shows that transaction costs have only a second-order effect on the liquidity premium implied by equilibrium asset returns *since* investors are able to accommodate large transaction costs by drastically reducing the frequency and volume of trade. Aiyagari and Gertler (1990), Vayanos (1998) and Vayanos and Villa (1998) obtain similar results.

The results in Huang (2003) show that some securities have high liquidity premium despite low turnover frequency. Huang (2003) achieves this by considering a more realistic assumption that investors are constrained from borrowing against future income shocks. He shows that the randomness of the holding horizon (i.e. liquidity risk) has a large effect on the equilibrium liquidity premium in an economy with borrowing constraints.

Acharya and Pedersen (2005) investigate the various channels through which liquidity risk may affect asset prices and find that a security's required return depends on its expected liquidity as well as on the covariances of its own return and liquidity with the market return and liquidity. Furthermore, they empirically find that the major part of the effect of liquidity risk arises from the covariance between the security liquidity and market return.

As for bond markets, few theoretical models have been developed to examine how liquidity and liquidity risk affect the expected returns of non-defaultable and defaultable bonds. Boudhoukh and Whitelaw (1993), motivated by the fact that almost identical Japanese government bonds can trade at large price differentials, study the issue of the value of liquidity in markets for non-defaultable bonds. They show that the observed price segmentation is the consequence of the liquidity differentials of bonds provided by the issuers who are able to charge for the liquidity services provided.

Tychon and Vannetelbosch (2005) develop a binomial bond valuation model that takes into account both the risk of early default and liquidity risk. They use a strategic bargaining setup in which transactions take place because investors have heterogeneous prior beliefs about bankruptcy costs. They show that liquidity risk can generate large yield spreads and the rich dynamics of the term structure of yield spreads. The *on-the-run* phenomenon, which has been documented in the empirical workings like Sarig and Warga (1989), can be explained by this model.

Ericsson and Renault (2007) is the first model which takes into account liquidity risk in the structural bond pricing framework. They extend Fan and Sundaresan (2000) by incorporating the influence of the liquidity risk of the market for distressed bonds on the renegotiation in financial distress. For some parameter inputs, substantial yield spreads are generated even for short term bonds. In addition, they find that levels of liquidity spreads are likely to be positively correlated with credit risk.

Empirically, the effects of liquidity and liquidity risk on asset returns have been intensively explored as well. Whether liquidity is priced in securities depends on the fact of whether it is a systematic risk. If it is not systematic, we can diversify it and then there will be no liquidity premia. For stock markets, Chordia et al. (2000), Huberman and Halka (2001) and Pastor and Stambaugh (2003) document the existence of common factors of liquidity[3]. Amihud (2002) adopts the liquidity measure of 'ILLIQ' to show that over time, expected market liquidity positively affects ex ant stock excess returns, suggesting that expected stock excess return partly represents a liquidity premium[4].

As for bond markets, Amihud and Mendelson (1991), Goldreich et al. (2003), Longstaff (2001) and Wang et al. (2005) study the effects of liquidity on U.S. Treasury bonds. They

[3]Note that the common factors documented in Pastor and Stambaugh (2003) are less significant

[4]This so-called ILLIQ measure for stock i in month t is estimated as follows:

$$ILLIQ_{i,t} = \frac{1}{D_t} \sum_{d=1}^{D_t} \frac{\left| r_{i,t}^d \right|}{V_{i,t}^d}$$

where D_t denotes the number of trading days in month t, $r_{i,t}^d$ denotes the return on the stock i in the d^{th} day of month t, and $V_{i,t}^d$ denotes the dollar trading volume for stock i in the d^{th} day day of month t, as a percentage of the dollar market capitalization of the stock. The basic intuition is to measure the daily price impact of order flow: the more liquid a stock, the smaller the ILLIQ measure.

show that the yields to maturities on notes, Refcorp bonds[5] and *off-the-run* Treasury securities are respectively higher than those on bills, Government bonds and *on-the-run* Treasury securities. They attribute this to the fact that notes, Refcorp bonds and *off-the-run* Treasury securities are respectively less liquid than bills, Government bonds and *on-the-run* Treasury securities.

Ericsson et al. (2006), Huang and Huang (2003) and Longstaff et al. (2004) try to answer one question: how much of the corporate yield spreads is due to credit risk? Ericsson et al. (2006) and Longstaff et al. (2004) use the data on the prices of Credit Default Swaps (CDS) which are commonly thought to be less influenced by non-default factors, while Huang and Huang (2003) use the data on default probabilities and loss rates provided by the rating agencies such as Moody's and Standard and Poor's. Their common observation is that only part of corporate yield spreads can be attributed to default risk and the remaining is due to other factors. Liquidity premium is shown to be an important non-default component.

De Jong and Driessen (2005) and Chacko (2006) directly study liquidity premia in corporate bond markets. With data from both U.S. and European corporate bond markets for the period from 1993 to 2002, De Jong and Driessen (2005) find that liquidity is a priced factor for the expected returns on corporate bonds. In terms of expected returns, the total estimated liquidity premium is around 45 basis points for investment-grade bonds with long maturity. For speculative-grade (*'junk'*) bonds, which have higher exposures to liquidity factors, liquidity premium is about 1%. Chacko (2006) adopts a new liquidity measure *latent liquidity*[6] which measures the turnover frequency (or accessibility) of corporate bonds, he uses data from the National Association of Insurance Commissioners (NAIC) in U.S. corporate bond markets. His results show that liquidity is not only important in explaining returns, but more importantly that it is also priced.

[5]Refcorp bonds are issued by the Resolution Funding Corporation, a government agency created by the Financial Institutions Reforms, Recovery, and Enforcement act of 1989 (FIRREA). Refcorp bonds literally have the same credit risk as Government bonds for their principal is fully collateralized by Government bonds

[6]For a securities dealer, what really determines the liquidity of a security is the ease in which a dealer can access a security. If a bond is more accessible by dealers, we can say that it is more liquid. The turnover frequency measures exactly how accessible a security is: the higher turnover frequency, the more accessible. The measure of latent liquidity can help us overcome the difficulty of extremely low trading activity in bond markets.

1.3 The Model

As mentioned above, we wish to extend the structural model of Leland and Toft (1996) to incorporate liquidity risk. An advantage of this model is that it generates closed-form solutions for equity value, debt value and firm value. It can be seen later on that closed-form solutions are still available even when liquidity risk is included.

This section builds a continuous-time valuation framework for illiquid bonds. Basic assumptions are listed and discussed below, some of which paralleling those in Merton (1974), Black and Cox (1976), Brennan and Schwartz (1978) and Leland and Loft (1996).

ASSUMPTION 1: *The firm has productive assets whose unleveraged value V follows a continuous diffusion process with constant proportional volatility* σ_1*:*

$$\frac{dV}{V} = (\mu(V,t) - \delta)\, dt + \sigma_1 dW_t^v \tag{1.1}$$

where $\mu(V,t)$ *is the total expected rate of return on asset value V;* δ *is the constant fraction of value paid out to security holders; and* dW_t^v *is the increment of a standard Brownian motion;*

ASSUMPTION 2: *As in Leland and Toft (1996), we assume that the debt structure of a firm is stationary;*

To be precise, this assumption means that a firm continuously issues certain amount of new bonds (principal) with maturity T (years) from issuance when the same amount of principal is retired such that its total understanding bond principal, denoted by P, is fixed over time. P is uniformly distributed over maturities in the time interval [0, T], so new bond principal is issued at a rate p=P/T per year. While the total annual coupon rate is C, bond with principal p continuously pays a constant coupon rate c=C/T per year. The total bond service payments are therefore time-dependent and equal C+P/T per year.

ASSUMPTION 3: *The firm will be forced to bankruptcy as soon as the firm value V falls below the bankruptcy-triggering value* V_B*, which is assumed to be constant and will be endogenously determined;*

In practice, the firm will not be immediately bankrupt in financial distress: either equity holders have an out-of-court negotiation with bond holders or firm enters into court-supervised proceedings (Chapter 11 of the U.S. Bankruptcy Code). If out-of-court negotiation is successful or firm survives court-supervised proceedings, then bankruptcy can be avoided. Fan and Sundaresan (2000) have a detailed discussion about how this affects the optimal capital structure and dividend policy of a firm.

ASSUMPTION 4: *The strict absolute priority holds. When bankruptcy occurs, the remaining value is distributed to bondholders and equityholders receive nothing. The fraction of firm asset value lost in bankruptcy is α, which is exogenously given in this paper. For simplicity, we assume that all bonds of different maturities from 0 to T have the same seniority over the remaining value in bankruptcy.*

ASSUMPTION 5: *The risk-free interest rate, denoted by r, is constant over time;*

This assumption is made for convenience. Shimko et al. (1993), and Longstaff and Schwartz (1995) discuss how stochastic interest rates affect the pricing of risky debts.

When liquidity risk is considered later in this paper, the condition that financial markets are complete and frictionless does not hold any more since liquidity, as mentioned above, is associated with transaction costs and private information. In this case, risk neutral valuation principle does not apply. In order to solve this problem, we assume that investors are risk neutral. This assumption makes us ignore the market risk premium required by investors, which may be important in explaining the yield spread puzzle, however it should not affect our results.

ASSUMPTION 6: *Investors are risk neutral.*

1.3.1 Without Liquidity Risk

Consider a perfectly liquid bond issuance with maturity t, which continuously pays a constant coupon flow c and has principal p, let $\rho = (1-\alpha)/T$ denote the fraction of asset value V_B^L which remains for the holders of this bond in bankruptcy, where index L denotes the fact that the bond is perfectly liquid (the same hereinafter). Using risk-neutral valuation and letting $f(s, V, V_B^L)$ denote the density of the first passage time s to V_B^L from V when the drift rate is $(r-\delta)$ gives the value of risky bond with maturity t as:

$$\begin{aligned} d^L\left(V, V_B^L, t\right) = & \int_0^t e^{-rs} c\left(1 - F\left(s, V, V_B^L\right)\right) ds \\ & + \int_0^t e^{-rs} \rho V_B^L f\left(s; V, V_B^L\right) ds + e^{-rt} p\left(1 - F\left(t; V, V_B^L\right)\right) \end{aligned} \tag{1.2}$$

$F(s)$ is the cumulative distribution function of the first passage time to bankruptcy. The first and third terms in Eq. (1.2) represents the expected discounted values of the coupon flow and principal if no default, the second term represents the expected discounted value of the fraction of the assets which will go to bond with maturity t, if bankruptcy occurs. Integrating the first and second terms by parts and using the results from Harrison (1990) and Rubinstein and Reiner (1991) give:

$$d^L(V; V_B^L, t) = \frac{c}{r} + e^{-rt}\left[p - \frac{c}{r}\right][1 - F(t)] + \left[\rho V_B^L - \frac{c}{r}\right]G(t) \tag{1.3}$$

where

$$F(t) = N(h_1(t)) + \left(\frac{V}{V_B^L}\right)^{-2a} N(h_2(t)) \tag{1.4}$$

$$G(t) = \left(\frac{V}{V_B^L}\right)^{-a+z} N(q_1(t)) + \left(\frac{V}{V_B^L}\right)^{-a-z} N(q_2(t)) \tag{1.5}$$

where a, b, z, $q_1(t)$ and $q_2(t)$ can be obtained by replacing V_B^I with V_B^L in the equations for a^I, b^I, z^I, $q_1^I(t)$ and $q_2^I(t)$ in the Appendix in Section 1.6, and

$$h_1(t) = \frac{(-b - a\sigma_1^2 t)}{\sigma_1\sqrt{t}}; \quad h_2(t) = \frac{(-b + a\sigma_1^2 t)}{\sigma_1\sqrt{t}} \tag{1.6}$$

With the assumptions of a stationary capital structure and strict absolute priority, the value of all outstanding bonds, when maturity of newly issued bonds is T, is determined as:

$$\begin{aligned} D^L\left(V; V_B^L, t\right) &= \int_0^T d^L(V; V_B^L, t)dt \\ &= \frac{C}{T} + \left[P - \frac{C}{r}\right]\left[\frac{1 - e^{-rT}}{rT} - I(T)\right] + \left[(1-\alpha)V_B^L - \frac{C}{r}\right]J(T) \end{aligned} \tag{1.7}$$

where

$$I(T) = \frac{1}{rT}\left[G(T) - e^{-rT}F(T)\right] \tag{1.8}$$

$$J(T) = \frac{1}{z\sigma_1\sqrt{T}}\left[-\left(\frac{V}{V_B^L}\right)^{-a+z} N(q_1(T))q_1(T) + \left(\frac{V}{V_B^L}\right)^{-a-z} N(q_2(T))q_2(T)\right] \tag{1.9}$$

As in Leland (1994), the total firm value is given by:

$$v^L(V; V_B^L) = V + \frac{\tau C}{r}\left[1 - \left(\frac{V}{V_B^L}\right)^{-(a+z)}\right] - \alpha V_B^L\left(\frac{V}{V_B^L}\right)^{-(a+z)} \tag{1.10}$$

where τ is the tax rate. In order to determine the endogenous bankruptcy-triggering value V_B^L, we invoke the smooth-pasting condition, V_B^L solves the following equation:

$$\frac{\partial E^L(V; V_B^L, T)}{\partial V}|_{V=V_B^L} = 0 \tag{1.11}$$

where $E^L(V; V_B^L, T)$ is equity value and equals the difference between total firm value $v^L(V; V_B^L)$ and the value of bond $D^L(V; V_B^L, T)$. The solution to Eq. (1.11) is independent of time and equals:

$$V_B^L = \frac{\frac{C}{r}\left(\frac{A}{rT} - B\right) - \frac{AP}{rT} - \frac{\tau x C}{r}}{1 + \alpha(a+z) - (1-\alpha)B} \tag{1.12}$$

where

$$A = z - a + 2ae^{-rT}N(a\sigma_1\sqrt{T}) - 2zN(z\sigma_1\sqrt{T}) - \frac{2}{\sigma_1\sqrt{T}}\left[n(z\sigma_1\sqrt{T}) - e^{-rT}n(a\sigma_1\sqrt{T})\right] \tag{1.13}$$

$$B = z - a - \left[2z + \frac{2}{z\sigma_1^2 T}\right]N(z\sigma_1\sqrt{T}) - \frac{2}{\sigma_1\sqrt{T}}n(z\sigma_1\sqrt{T}) - \frac{1}{z\sigma_1^2 T} \tag{1.14}$$

The result is consistent with *ASSUMPTION 3* since V_B^L is constant with respect to maturity T. Using the above result of V_B^L in the equations (1.7) and (1.10) yields closed-form solutions for bond value and firm value in the case when there is no liquidity risk.

1.3.2 With Liquidity Risk

Leland and Toft (1996), along with other structural models, ignores an important risk factor: liquidity risk. As stated in Section 1.2, many theoretical and empirical papers have shown that liquidity risk is statistically and economically important in explaining asset returns.

To model liquidity risk, we now assume that bondholders may encounter liquidity shocks during the lifetime of corporate bonds. The reasons for liquidity shocks may be that either there are other better investment opportunities or bondholders just want to adjust their portfolio asset allocation. When liquidity shocks occur, investors sell their bonds at a discount price.

ASSUMPTION 7: *Bondholders may encounter liquidity shocks during the lifetime of the bonds and the probability of liquidity shocks follows a Poisson distribution of intensity λ per year;*

With Poisson distributed liquidity shocks, the bond-holding horizon of investors is unknown in advance and hence liquidity risk arises.

ASSUMPTION 8: *Bondholders will immediately sell the bonds they hold when they face liquidity shocks;*

In other words, this means that investors are constrained from borrowing when they face liquidity shocks. As explained by Huang (2003), this is a realistic assumption due to the randomness of the holding horizon. Technically, such an assumption is made for obtaining tractable closed-form solutions in this paper.

ASSUMPTION 9: *When a liquidity shock occurs at time s in the interval [0, t], bonds will be sold at a discounted price:* $\zeta(s)d^L(V_s;V_B^I,t-s)$, *where t-s is the time to maturity,* $0 < 1-\zeta(s) < 1$ *is the discount fraction at time s and* $d^L(V_s;V_B^I,t-s)$ *is the value of identical but perfectly liquid bonds with initial value* V_s *and maturity t-s.*[7].

In the traditional structural models, the cash flows to bonds of maturity t have three sources: coupon rate until the maturity if the issuer is always solvent before the maturity or until the time when the issuer is bankrupt before the maturity, the principal if the issuer survives until the maturity or the remaining value of firm which can be distributed to bondholders in bankruptcy if the issuer is bankrupt before the maturity. When liquidity risk is incorporated, the sources of cash flows to bonds of maturity t change: first, if there is neither bankruptcy nor a liquidity shock before bond maturity, then bondholders will receive a continuous coupon c until t and principal p at maturity; second, if bankruptcy takes place before liquidity shock and maturity, bondholders will continuously receive coupon c until bankruptcy time when they will also receive the remaining asset value of firm ρV_B^I in bankruptcy; third, if a liquidity shock takes place before default and maturity, bondholders will continuously receive coupon c until the time of the liquidity shock when they will sell bonds at a discount price $\zeta(s)d^L(V_s;V_B^I,t-s)$. Therefore, mathematically, the presence of liquidity risk makes the value of bonds of maturity t become:

$$
\begin{aligned}
d^I\left(V;V_B^I,t\right) = & \int_0^t e^{-rs}c\left(1-F\left(s;V,V_B^I\right)\right)Pr\left(N\left[0,s\right]=0\right)ds \\
& + \int_0^t e^{-rs}\rho V_B^I f\left(s;V,V_B^I\right)Pr\left(N\left[0,s\right]=0\right)ds \\
& + E\left[\int_0^{t^-} e^{-rs}\left(\zeta(s)d^L(V_s;V_B^I,t-s)\right)\left(1-F\left(s;V,V_B^I\right)\right)\lambda e^{-\lambda s}ds\right] \\
& + e^{-rt}p\left(1-F\left(t;V,V_B^I\right)\right)Pr\left(N\left[0,t\right]=0\right) \qquad (1.15)
\end{aligned}
$$

where $Pr(N[0,s]=0)$ is the probability that there is no liquidity shock until time s and

[7] Index I means that bonds are not perfectly liquid or illiquid (the same hereinafter)

$\lambda e^{-\lambda s}$ the density function of the first liquidity shock time[8]. With the results in the Appendix in Section 1.6, the value of illiquid risky bonds is simplified into:

$$\begin{aligned} d^I(V;V_B^I,t) = & \frac{c}{r+\lambda} + e^{-(r+\lambda)t}\left[p - \frac{c}{r+\lambda}\right][1-F(t)] \\ & + e^{-\frac{b^I(r-r^I)}{\sigma_1^2}}\left[\rho V_B^I - \frac{c}{r+\lambda}\right]G^I(t) \\ & + \lambda E\left[\int_0^{t-} e^{-(r+\lambda)s}\zeta(s)d^L(V_s;V_B^I,t-s)(1-F(s))ds\right] \end{aligned} \tag{1.16}$$

where r^I and b^I are defined in the Appendix in Section 1.6; F(t) is the same as in Section 1.3.1; $G^I(t)$ is similar to G(t) except that the riskless interest rate r and b in the expression for G(t) are now replaced by r^I and b^I.

1.3.2.1 Case 1: Constant $\zeta(t)$

We first assume that $\zeta(t)$ is a constant and we denote it by $\overline{\zeta}$, this assumption will be relaxed in the next subsection to check whether the variance of $\zeta(t)$ and its correlation with firm value influence optimal capital structure and yield spreads. The value of all outstanding illiquid risky bonds, when bond of maturity T is issued, is determined as:

$$\begin{aligned} D^I(V;V_B^I,T) = & \int_0^T d^I(V;V_B^I,t)dt \\ = & \frac{C}{r+\lambda} + \left[P - \frac{C}{r+\lambda}\right]\left[\frac{1-e^{-(r+\lambda)T}}{(r+\lambda)T} - I(T)\right] \\ & + e^{-\frac{b^I(r-r^I)}{\sigma_1^2}}\left[(1-\alpha)V_B^I - \frac{C}{r+\lambda}\right]J^I(T) \\ & + \lambda\overline{\zeta}E\left[\int_0^T\int_0^{t-} e^{-(r+\lambda)s}d^L(V_s;V_B^I,t-s)[1-F(s)]dsdt\right] \end{aligned} \tag{1.17}$$

The presence of liquidity risk will only affect total firm value by changing the endogenous bankruptcy triggering level from V_B^L into V_B^I. Therefore, total firm value becomes:

$$v^I(V;V_B^I,T) = V + \frac{\tau C}{r}\left[1 - \left(\frac{V}{V_B^I}\right)^{-(a+z)}\right] - \alpha V_B^I\left(\frac{V}{V_B^I}\right)^{-(a+z)} \tag{1.18}$$

[8]The probability that the first liquidity shock takes place before time s (including s) is 1-Pr(N[0,s] = 0), taking derivative with respect to time s yields the intensity.

The equity value is equal to total firm value minus the value of all outstanding illiquid risky bonds:

$$E^I(V;V_B^I,T) = V^I(V;V_B^I,T) - D^I(V;V_B^I,T) \tag{1.19}$$

Again, using the smooth pasting condition, we obtain the optimal endogenous bankruptcy triggering value V_B^I as follows:

$$V_B^I = \frac{A^I - E\left(\int_0^T \int_0^{t^-} \omega(s) M(W_s^v,t,s) ds dt\right)}{B^I + \rho E\left(\int_0^T \int_0^{t^-} \omega(s) N(W_s^v,t,s) ds dt\right)} \tag{1.20}$$

where $\omega(s) \; = \; \lambda\overline{\zeta}e^{-(r+\lambda)s}$

$$\begin{aligned} A^I \; = \; & \frac{\tau(a+z)C}{r} - \frac{(r-r^I)C}{(r+\lambda)\sigma_1^2} + \frac{1}{rT}\left[P - \frac{C}{r+\lambda}\right] \\ & \times \left\{ z - a - 2z\Phi(z\sigma_1\sqrt{T}) - \frac{2\phi(z\sigma_1\sqrt{T})}{\sigma_1\sqrt{T}} + e^{-rT}\left[2a\Phi(a\sigma_1\sqrt{T}) + \frac{2\phi(a\sigma_1\sqrt{T})}{\sigma_1\sqrt{T}}\right]\right\} \\ & - \frac{C}{(r+\lambda)}\left[a^I - z^I + 2z^I\Phi(z^I\sigma_1\sqrt{T}) + \frac{2\phi(z^I\sigma_1\sqrt{T})}{\sigma_1\sqrt{T}} + \frac{2\Phi(z^I\sigma_1\sqrt{T})-1}{z^I\sigma_1^2 T}\right] \end{aligned} \tag{1.21}$$

$$\begin{aligned} B^I \; = \; & -1 - \alpha(a+z) - \frac{(r-r^I)(1-\alpha)}{\sigma_1^2} - (1-\alpha) \\ & \times \left[(a^I - z^I) + 2z^I\Phi(z^I\sigma_1\sqrt{T}) + \frac{2\phi(z^I\sigma_1\sqrt{T})}{\sigma_1\sqrt{T}} + \frac{2\Phi(z^I\sigma_1\sqrt{T})-1}{z^I\sigma_1^2 T}\right] \end{aligned} \tag{1.22}$$

$$\begin{aligned} M(W_s^v,t,s) \; = \; & \frac{c}{r}\left(1 - \left(\Phi(q_1^*(t-s)) + \Phi(q_2^*(t-s))\right)\right)\left[2a\Phi(a\sigma_1\sqrt{s}) + \frac{2\phi(a\sigma_1\sqrt{s})}{\sigma_1\sqrt{s}}\right] + e^{-r(t-s)} \\ & \times \left(p - \frac{c}{r}\right)\left(1 - \left(\Phi(h_1^*(t-s)) + \Phi(h_2^*(t-s))\right)\right)\left[2a\Phi(a\sigma_1\sqrt{s}) + \frac{2\phi(a\sigma_1\sqrt{s})}{\sigma_1\sqrt{s}}\right] \end{aligned} \tag{1.23}$$

$$N(W_s^v,t,s) \; = \; \left(\Phi(q_1^*(t-s)) + \Phi(q_2^*(t-s))\right)\left[2a\Phi(a\sigma_1\sqrt{s}) + \frac{2\phi(a\sigma_1\sqrt{s})}{\sigma_1\sqrt{s}}\right] \tag{1.24}$$

where a^I and z^I are defined in the Appendix in Section 1.6, $h_{1,2}^*(t-s)$ and $q_{1,2}^*(t-s)$ are defined as follows:

$$h_{1,2}^*(t-s) \; = \; \frac{-\left(\left(r-\delta-\frac{1}{2}\sigma_1^2\right)s + \sigma_1 W_s^v\right) \mp a\sigma_1^2(t-s)}{\sigma_1\sqrt{t-s}} \tag{1.25}$$

$$q_{1,2}^*(t-s) \; = \; \frac{-\left(\left(r-\delta-\frac{1}{2}\sigma_1^2\right)s + \sigma_1 W_s^v\right) \mp z\sigma_1^2(t-s)}{\sigma_1\sqrt{t-s}} \tag{1.26}$$

Note that V_B^I is independent of time again. Substituting V_B^I into the equations (1.17), (1.18), and (1.19) yields the values of equities and bonds and firm value.

1.3.2.2 Case 2: Time-Varying $\zeta(t)$

In practice, the assumption of constant $\zeta(t)$ is not realistic. For example, Fujimoto (2003) examines the macroeconomic sources of systematic liquidity and finds that various macroeconomic factors, including inflation and monetary policy, not only influence liquidity directly, but also indirectly through their effects on the market variables such as market return, volatility and share turnover that are found to be other important drivers of liquidity. Obviously, macroeconomic factors and market variables are not deterministic and they fluctuate over time. Therefore, liquidity should change over time as well. Pastor and Stambaugh (2003) and Acharya and Pedersen (2005) have documented that aggregate liquidity does change over time and liquidity tends to drop when the market is down and volatility is high.

In this paper, we assume that time-varying $\zeta(t)$ follows a mean-reverting stochastic process as:

$$d\zeta_t = \kappa_\zeta(\zeta_m - \zeta_t)dt + \sigma_2 dW_t^\zeta \tag{1.27}$$

where ζ_m is the long-term value of $\zeta(t)$; κ_ζ is the mean-reverting speed; σ_2 is the constant volatility of $\zeta(t)$ and $dW_t^v dW_t^\zeta = \rho_1$ ($\rho_1 \neq 0$). A nonzero correlation coefficient ρ_1 allows us to incorporate the influence of the overall state of the economy on both a firm's asset value and $\zeta(t)$. For example, if $\rho_1 > 0$, then during economic recessions, firm value would tend to decrease while discount factor $1 - \zeta(t)$ increases.

With such a time-varying $\zeta(t)$, the values of equities and bonds and firm value are similar to those in the case of constant $\overline{\zeta}$ except that $\omega(s)$ changes into:

$$\omega(s) = \lambda e^{-(r+\lambda)s}\left[(\zeta_0 - \zeta_m)e^{-\kappa_\zeta s} + \zeta_m + \sigma_2 \int_0^s e^{\kappa_\zeta(u-s)} dW_u^\zeta\right] \tag{1.28}$$

1.4 Numerical Results

This section summarizes the numerical results of how liquidity risk affects the optimal capital structure of a firm and yield spreads of corporate bonds. The base-case scenario parameters chosen here are: $V = 100$, $r = 0.06$, $\delta = 0.07$, $\sigma_1 = 0.20$, $\alpha = 0.50$, $\tau = 0.35$, $\lambda = 0.5$

[9], $\overline{\zeta} = 0.9973$[10]. The coupon rate c is exogenously given and equals 0.075 per year for one dollar principal.

For each firm, the principal of bonds is chosen such that firm value is maximized. To determine the endogenously determined bankruptcy-triggering value V_B^I, which will be used to compute the bond value D^I, equity value E^I and firm value v^I, we use Monte Carlo method to simulate and calculate the expected terms in the expression for V_B^I. For each maturity T, we simulate 2,000 standard normal random variables and calculate the double integral for each simulation and then take average. The simulation method can also be adopted in calculating d^I and D^I, while the numbers of simulation increase to 100,000 for the purpose of ensuring stable results. For maturities less than 20 years, simulated results are quite stable. The yield to newly issued bond of maturity T is obtained implicitly through

$$d^I(V; V_B^I, T) = \int_0^T e^{-Rt} c dt + e^{-RT} p \tag{1.29}$$

and the yield spread is defined via:

$$YS = R - r \tag{1.30}$$

1.4.1 Optimal Capital Structure

When liquidity shock density $\lambda = 0.5$ and $\zeta(t)$ is constant: $\overline{\zeta} = 0.9973$, firm value is plotted in Figure 1 as a function of leverage ratio (D/v and D^I/v^I) for firms issuing bonds with maturities T which are respectively 0.5, 5, 20 in Panel A, B and C. The long dashed lines represents functions of firm values for model without liquidity risk, and the solid lines represents functions of firm values for model with liquidity risk. For those three different maturities, firm value is always smaller when there is liquidity risk and the difference increases with maturity. Firm value increases with maturity if we do not take into account liquidity risk, however, this does not hold any more as long as liquidity risk is incorporated into the structural model. With liquidity risk, firm value first increases from 103.1 to 105.4 when the maturity of bonds increases from 6 months to 7.5 years and then decreases to 103.27 when maturity further increases to 20 years. As indicated in Table 1, when considering liquidity risk, it is optimal for firms to issue bonds whose maturity is around 7.5 years (less than 10 years) but not 20 years since firm value is maximized (around 105) with such bonds.

[9] Chacko (2005) observes that the median age of trading corporate bonds is 4.3 years in 2004. Considering the fact that corporate bonds can be repeatedly traded, we take $\lambda = 0.5$.

[10] Schultz (2001) empirically finds that average corporate bond trading costs are about \$0.27 per \$100 of par value.

This result is consistent with empirical finding. As stated in the introduction, the average maturity of corporate bonds has been declining since the 1990's and was around 7 years in 2005. However, Leland and Toft (1996) shows that the longer the bond maturity, the larger the firm value, and therefore firms would prefer to issue bonds of possible long maturity. The well accepted reason for firms not to issue many bonds with long maturity is that there exist agency conflicts between shareholders and bond holders and short-term bonds can be used to limit shareholders to deviate for riskier projects. In this paper, we provide another possibility to explain why firms on average do not issue long-term bonds: the longer the bond maturity, the higher the probability that bondholders will encounter liquidity shock during the lifetime of the bonds, and hence bondholders will require higher returns on long-term bonds to compensate for costs associated with liquidity shock. This will raise costs from issuing bonds of long maturity and then reduce firm value.

Similarly, Figure 1 also shows that the relationship between maturity and optimal leverage ratio is not monotonic. As seen in Table 1, the optimal leverage ratio with liquidity risk, which maximizes firm value over time, is around 23%. During the 1990's, the average leverage ratio was approximately 30% in the United States and even lower in some other countries like Canada or Singapore. Recently, the average market leverage ratio of S&P500 index firms has declined to below 15%. The reason why the leverage ratio of a firm is low can be found in Figure 2, which plots the value of all outstanding bonds as a function of leverage ratio for different issuance maturities T which are respectively equal to 0.5, 5 and 20 years in Panel A, B and C under both cases without and with liquidity risk. The bond capacity (value) is clearly reduced when there exists liquidity risk for corporate bonds. When firm value is maximized, bond capacity is approximately 24 with liquidity risk and 54 without liquidity risk. The big difference between bond capacities makes optimal leverage ratios distinct.

When liquidity shock density λ increases to 2 and $\overline{\zeta}$ remains unchanged, the optimal leverage ratio of a firm decreases by 4%, this is shown in Table 2. Obviously, when λ changes from 0.5 to 2, the probability that liquidity shocks take place increases, therefore the costs from issuing bonds are higher.

Now suppose that $\zeta(t)$ is stochastic and follows the process defined in Eq. (1.27) and λ again equals 0.5. It is implicitly embedded in Eq. (1.27) that $\zeta(t)$ could be greater than 1, this, according to the definition of $\zeta(t)$, is impossible. When simulating $\zeta(t)$, we limit it to the range between 0.99 and 1. The lower bound is chosen since transaction costs are rarely more than 1% in practice. Whether do the variance of $\zeta(t)$ and its correlation with firm value affect the optimal capital structure of a firm? The results can be found in Table 3 which compares the characteristics of optimally leveraged firms under different combinations of σ_2 and ρ_1. For the sake of ease comparison, we assume that both ζ_0 and ζ_m are equal to 0.9973. As shown in Panel A, when $\sigma_2 = 0.1$ and $\rho_1 = 0.2$, the variance of $\zeta(t)$ and its correlation with firm value almost have no effect on the optimal capital structure of a firm. The conclusion holds even when the variance and the correlation increase to some high levels, this is shown in Panels B and C where σ_2 (or ρ_1) increases to 0.3 (or 0.6).

1.4.2 Yield Spreads

First, consider the base-case scenario with $\lambda = 0.5$ and $\bar{\zeta} = 0.9973$. Figure 3 plots the yield spreads of newly issued investment-grade bonds as a function of maturities T[11]. For liquid bonds, yield spreads are almost negligible when maturities are less than 4 years and then become significant when maturities are enough long. However, if we take into account liquidity risk, yield spreads are not negligible any more even for short-term bonds. For illiquid bonds of 6 months, yield spread is approximately 12 basis points, much higher than zero implied by Leland and Toft (1996). Interestingly, liquidity yield spreads decrease with maturities[12]. The possible reason for this phenomenon is: given that liquidity shocks have taken place, the expected time of liquidity shocks is shorter for short-term bonds and then the discounted liquidity costs will be larger. When maturity increases, credit risk will become more important and gradually dominate over liquidity risk. As a consequence, the relative size of liquidity yield spreads decreases with maturity.

The yield spreads for newly issued $'junk'$ bonds are shown in Figure 4. In the case of $'junk'$ bonds, liquidity risk also increases yield spreads. For $'junk'$ bonds with short-term maturities, the relative size of liquidity yield spreads is small. When investing in short-term $'junk'$ bonds, investors mostly worry about credit risk and hence liquidity yield spreads are low. When the maturity of $'junk'$ bonds increases, investors become very sensitive to liquidity shocks. To keep the leverage ratio of 60%, a firm has to issue bonds with lower prices, therefore yield spreads significantly increase. Liquidity yield spread is maximized when maturity is around 11 years and then decreases with maturity. Liquidity risk contributes to

[11] We define investment-grade bonds as those bonds which are issued by firms with relatively low leverage ratios (20% in our case). In contrast, $'junk'$ (or speculative) bonds are those bonds which are issued by firms with relatively high leverage ratio (60% in our case). Collin-Dufresne and Goldstein (2001) uses a similar category method.

[12] Liquidity yield spread is defined as the difference between the yield spreads with and without liquidity risk. To be careful that liquidity yield spread is not equal to the expected discounted liquidity cost.

around 30% of total yield spreads for long-term (20 years) $'junk'$ bonds, with the absolute size being slightly more than 1%. For $'junk'$ bonds, credit risk always dominates over liquidity risk.

To have more insights about how liquidity risk affects yield spreads of corporate bonds, we present, in Table 4, the characteristics of firms with 20% and 60% leverage ratios. Whatever debt maturity is, V_B^I is always larger than V_B^L and the difference increases with maturity. Moreover, not like V_B^L, V_B^I is not monotonically decreasing. When debt maturity is longer than 7.5 years (or 5 years in the case of 60% leverage ratio), V_B^I starts to increase and is more than doubled compared with V_B^L when $T = 20$. It is now clear that liquidity risk greatly raises the bankruptcy-triggering value which moves up the default probability, so firms have to issue bonds in lower prices and yield spreads increase.

Figure 5 shows what will happen for the yield spreads of investment-grade bonds if we change liquidity shock density λ from 0.5 to 2. The mostly influenced bonds are those whose maturities are less than 5 years. For bonds which will expire in 6 months, yield spread is almost tripled. Intuitively, to increase λ will raise the probability that liquidity shocks take place within short periods. When maturity becomes enough long, liquidity shocks will take place almost surely regardless of λ if it is not too small, hence the effect of increasing λ is very limited.

For $\lambda = 2$, the leverage ratio of a firm with positive equity value will not be as high as 60% when its bond maturity is long since the costs from issuing such bonds are too high.

Now suppose that $\zeta(t)$ is stochastic and follows the process defined in equation (27) and λ again equals 0.5. As above, we limit $\zeta(t)$ to the range between 0.99 and 1 and take $\zeta_0 = \zeta_m = 0.9973$. Figure 6 and Figure 7 show the effects of the variance of $\zeta(t)$ and its correlation with firm value on the yield spreads of investment-grade bonds.

In Figure 6, ρ_1 is fixed to 0.2 and σ_2 varies among 0, 0.1 and 0.3. First, we note that the term structure of yield spreads, which initially decrease and then increase with maturities, does not change. The explanation above works here as well, that is, given that liquidity shocks have taken place, the expected time of liquidity shocks is shorter for short-term bonds and then the discounted liquidity costs will be larger. Second, yield spreads for $\sigma_2 = 0.1, 0.3$ are very close to each other but higher than those for $\sigma_2 = 0$. However, it is worth noting that the yield spread difference is not due to the variation of σ_2. When $\sigma_2 = 0$, the expected value of $\zeta(t)$ equals 0.9973 and is higher than the expected $\zeta(t)s$ obtained in the case of $\sigma_2 = 0.1, 0.3$ because of the asymmetric lower and upper bounds around the mean of $\zeta(t)$. It is likely that different expected discount factors generate the yield spread difference.

The variance of $\zeta(t)$ has therefore rather weak effects on liquidity yield spreads. Similar results are shown in Figure 7 where $\sigma_2 = 0.1$ and ρ_1 varies among 0, 0.2 and 0.6, hence the correlation between $\zeta(t)$ and firm value almost does not affect liquidity yield spreads neither.

As $'junk'$ bonds, the effects of the variance of $\zeta(t)$ and its correlation with firm value on yield spreads are shown in Figures 8 and 9 in which we find that both σ_2 and ρ_1 have little effects. The results are similar as in the case of investment-grade bonds.

Compared with Duffie and Lando (2001) and Collin-Dufresne and Goldstein (2001), our model is better since it generates yield spreads, which are significantly different from zero, for short-term investment-grade bonds. Although Zhou (2001) has better results than us. This assumption that the values of investment-grade firms may jump a lot within very short time like 6 months is however not that much realistic.

1.5 Conclusion

In this paper, we extend the structural model of Leland and Toft (1996) by incorporating liquidity risk. We hope that incorporating liquidity risk into the structural model can help generate the capital structure and yield spreads which are more consistent with empirical findings.

We model liquidity risk by assuming that bond holders may encounter liquidity shock during the lifetime of corporate bonds and the probability of liquidity shock follows a Poisson distribution. When facing a liquidity shock, bond holders are assumed to immediately sell the bonds they hold at a discounted price, which is first assumed to be constant and then relaxed to follow a mean-reverting diffusion process.

Our results show that incorporating liquidity risk has indeed generated better capital structure and yield spread:

- It is optimal for firms to issue bonds with medium maturity (around 7-8 years) rather than with long maturity;

- The optimal leverage ratio implied by our model is approximately 23% and close to the actual one in the financial markets;

- With liquidity risk, the yield spread of short-term investment-grade bonds is not negligible any more. The relative size of liquidity spread for investment-grade bonds is decreasing with maturity;

- The yield spread of $'junk'$ bonds with liquidity risk is higher. For long-term $'junk'$ bonds, liquidity yield is slightly higher than 1%.

Despite that our results are very suggestive and more consistent with empirical findings, the yield spreads predicted in this model are still very low for both short- and long-term investment-grade bonds. For example, Amato and Remolona (2003) find that, during the period from 1997 to 2003, triple-A rated bonds with maturities from 1 to 3 years have an average yield spread of 49.50 basis points, with 88.82 basis points for the average yield spread of single-A rated bonds of similar maturities[13]. One way of improving our results is to take into account liquidity jumps which may significantly increase yield spreads for short-term bonds. In practice, liquidity often dries up during economic recession and financial crisis (like the Asian financial crisis in 1997). However, according to Zhou (2001), jump risk does not affect the yield spreads of long-term bonds very much. One important issue not considered in this paper is that investors can sell bonds any time before expiration, this may mitigate the effects of liquidity shocks.

[13]See Table 1 in Amato and Remolona (2003) for more details

1.6 Appendix

1.6.1 The Valuation of Illiquid Bonds of Maturity t

After integration by parts and simplification, the first term in Eq. (1.15) becomes:

$$\frac{c}{r+\lambda}\left[1-(1-F(t))\,e^{-(r+\lambda)t}-\int_0^t e^{-(r+\lambda)s}f(s)ds\right] \tag{1.31}$$

Harrison (1990) shows that the density function $f(s,V,V_B^I)$ of the first passage time s to V_B^I from V is:

$$\frac{b^I}{\sigma_1\sqrt{2\pi s^3}}exp\left[-\frac{1}{2}\left(\frac{b^I+(r-\delta-\frac{\sigma_1^2}{2})s}{\sigma_1\sqrt{s}}\right)^2\right] \tag{1.32}$$

where

$$b^I = ln\left(\frac{V}{V_B^I}\right); \tag{1.33}$$

With this density function at hand, the integral of the last term inside the bracket of Eq. (1.31) can be written:

$$\int_0^t e^{-(r+\lambda)s}f(s)ds = e^{-\frac{b^I(r-r^I)}{\sigma_1^2}}\int_0^t e^{-r^Is}f^I(s)ds \tag{1.34}$$

where $f^I(s)$ is

$$\frac{b^I}{\sigma_1\sqrt{2\pi s^3}}exp\left[-\frac{1}{2}\left(\frac{b^I+(r^I-\delta-\frac{\sigma_1^2}{2})s}{\sigma_1\sqrt{s}}\right)^2\right] \tag{1.35}$$

and

$$r^I = \frac{1}{2}\left[-(\sigma_1^2-2\delta)+\left((\sigma_1^2-2\delta)^2+4(r^2-2r\delta+r\sigma_1^2+2\lambda\sigma_1^2)\right)^{1/2}\right] \tag{1.36}$$

Directly applying the result of Eq. (8) in Leland and Toft (1996) gives:

$$\int_0^t e^{-r^Is}f^I(s)ds = \left[\left(\frac{V}{V_B^I}\right)^{-a^I+z^I}N[q_1^I(t)]+\left(\frac{V}{V_B^I}\right)^{-a^I-z^I}N[q_2^I(t)]\right] \tag{1.37}$$

where

$$a^I = \frac{\left(r^I - \delta - (\sigma_1^2/2)\right)}{\sigma_1^2};\ z^I = \frac{[(a^I\sigma_1^2)^2 + 2r^I\sigma_1^2]^{1/2}}{\sigma_1^2};\ q_{12}^I = \frac{\left(-b^I \mp z^I\sigma_1^2 t\right)}{\sigma_1\sqrt{t}} \tag{1.38}$$

Applying the same method as in calculating the first term, we obtain that the second term of Eq. (1.15) equals:

$$\rho V_B^I e^{-\frac{b^I(r-r^I)}{\sigma_1^2}} \left[\left(\frac{V}{V_B^I}\right)^{-a^I+z^I} N[q_1^I(t)] + \left(\frac{V}{V_B^I}\right)^{-a^I-z^I} N[q_2^I(t)]\right] \tag{1.39}$$

The third term of Eq. (1.15) remains unchanged:

$$E\left[\int_0^{t^-} e^{-rs}\left[\zeta(s)d^L(V, V_B^I, t-s) - C_s\right]\left[1 - F\left(s\right)\right]\lambda e^{-\lambda s} ds\right] \tag{1.40}$$

The fourth term of Eq. (1.15) is equal to:

$$e^{-(r+\lambda)t} p[1 - F(t)] \tag{1.41}$$

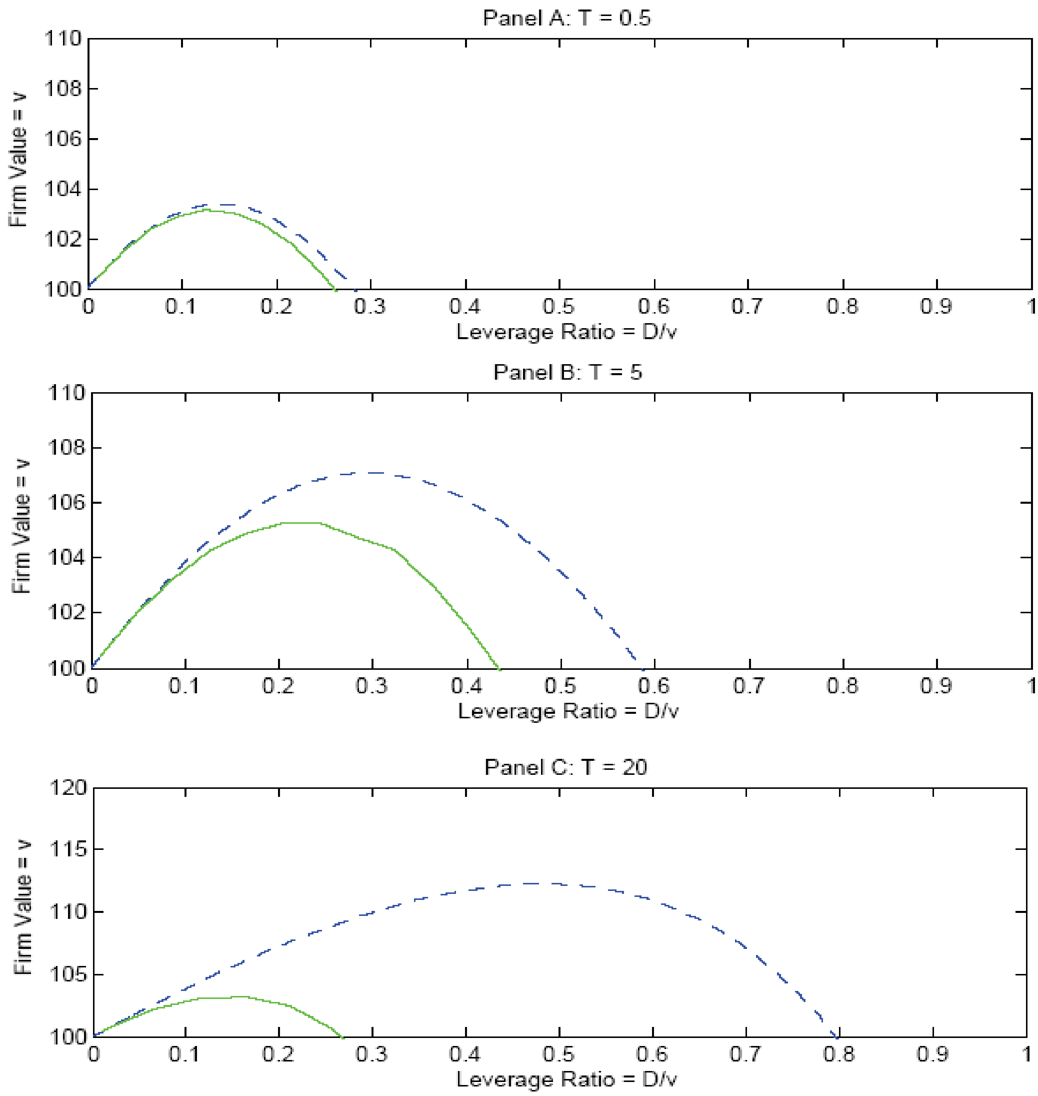

Figure 1: Firm values as a function of leverage ratios. The lines plot firm values as a function of leverage ratios for firms issuing debt with maturity T which is respectively 0.5, 5, 20 years in Panel A, B and C. The dashed lines represent firm values for the model without liquidity risk and the solid lines represents firm values for the model with liquidity risk. The parameters chosen here are: $V = 100$, $r = 0.06$, $\delta = 0.07$, $\sigma_1 = 0.20$, $\alpha = 0.50$, $\tau = 0.35$, $\lambda = 0.50$ and $\overline{\zeta} = 0.9973$. The coupon rate c is exogenously given and equal to 0.075 per year for one dollar principal.

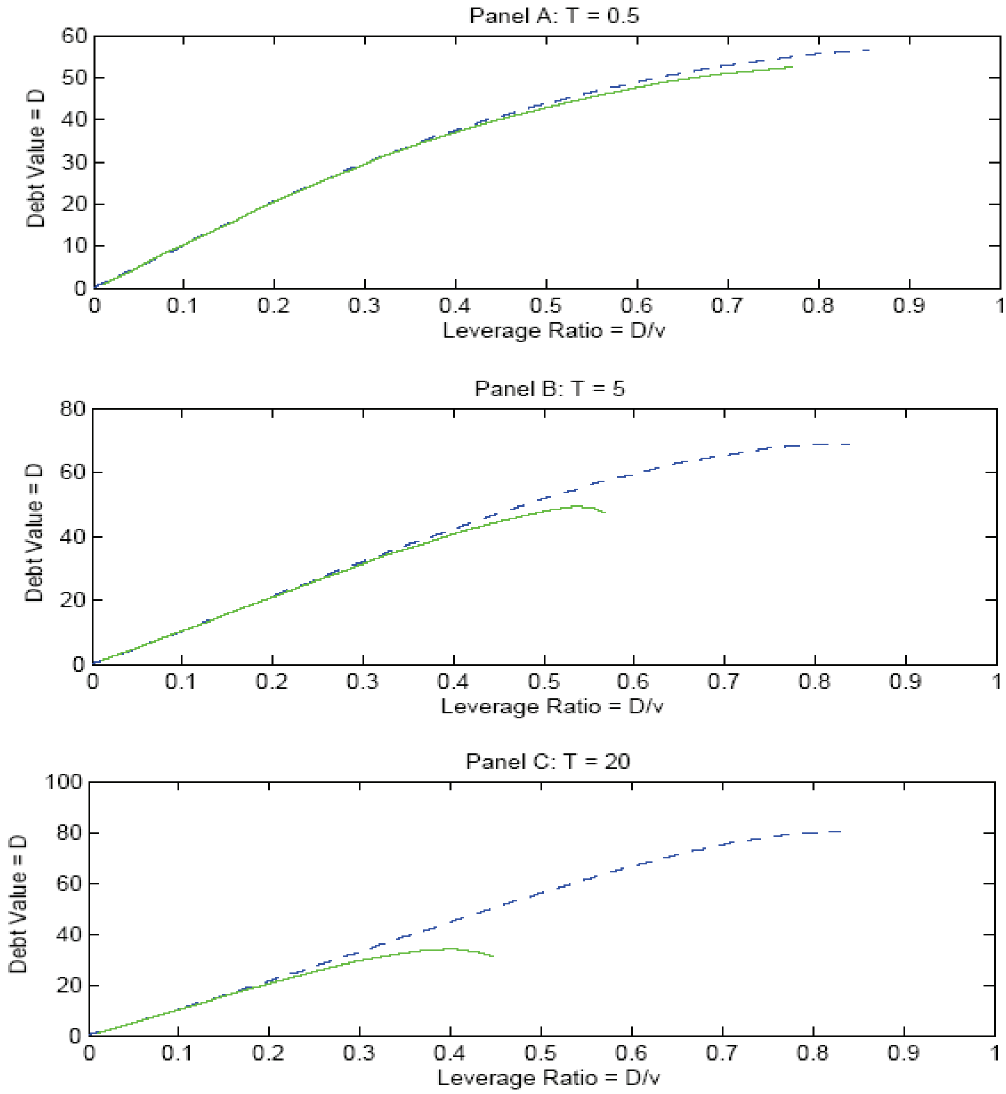

Figure 2: Debt values as a function of leverage ratios. The lines plot debt values as a function of leverage ratios for firms issuing debt with maturity T which is respectively 0.5, 5, 20 years in Panel A, B and C. The dashed lines represent debt values for the model without liquidity risk, and the solid lines represent debt values for the model with liquidity risk. The parameters chosen here are: $V = 100$, $r = 0.06$, $\delta = 0.07$, $\sigma_1 = 0.20$, $\alpha = 0.50$, $\tau = 0.35$, $\lambda = 0.50$ and $\overline{\zeta} = 0.9973$. The coupon rate c is exogenously given and equal to 0.075 per year for one dollar principal.

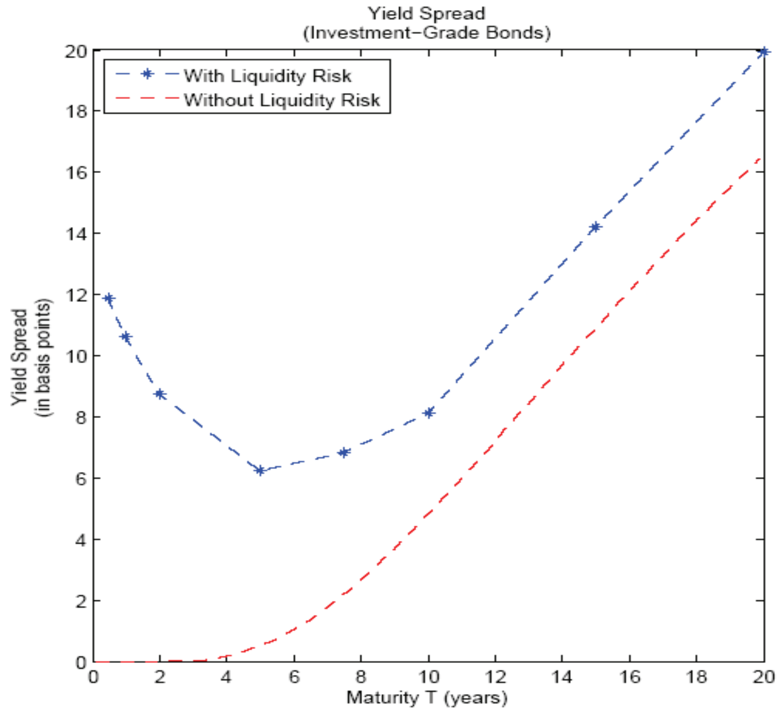

Figure 3: Yield spreads as a function of debt maturities. The lines plot yield spreads (in basis points) of newly issued bonds as a function of maturity T ranging from 1 month to 20 years for firms with leverage ratio 20 percent. The dashed line represents yield spreads for the model without liquidity risk, and the star line represents yield spreads for the model with liquidity risk. The parameters chosen here are: $V = 100$, $r = 0.06$, $\delta = 0.07$, $\sigma_1 = 0.20$, $\alpha = 0.50$, $\tau = 0.35$, $\lambda = 0.50$ and $\overline{\zeta} = 0.9973$. The coupon rate c is exogenously given and equal to 0.075 per year for one dollar principal.

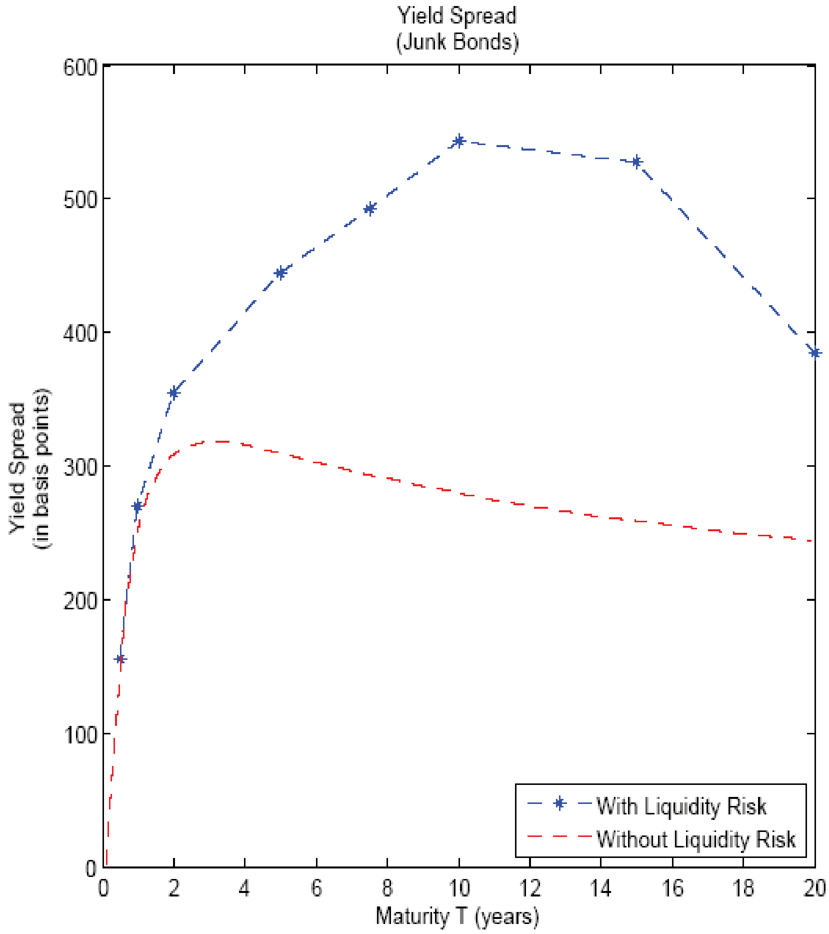

Figure 4: Yield spreads as a function of debt maturities. The lines plot yield spreads (in basis points) of newly issued bonds as a function of maturity T ranging from 1 month to 20 years for firms with leverage ratio 60 percent. The dashed line represents yield spreads for the model without liquidity risk, and the star line represents yield spreads for the model with liquidity risk. The parameters chosen here are: $V = 100$, $r = 0.06$, $\delta = 0.07$, $\sigma_1 = 0.20$, $\alpha = 0.50$, $\tau = 0.35$, $\lambda = 0.50$ and $\overline{\zeta} = 0.9973$. The coupon rate c is exogenously given and equal to 0.075 per year for one dollar principal.

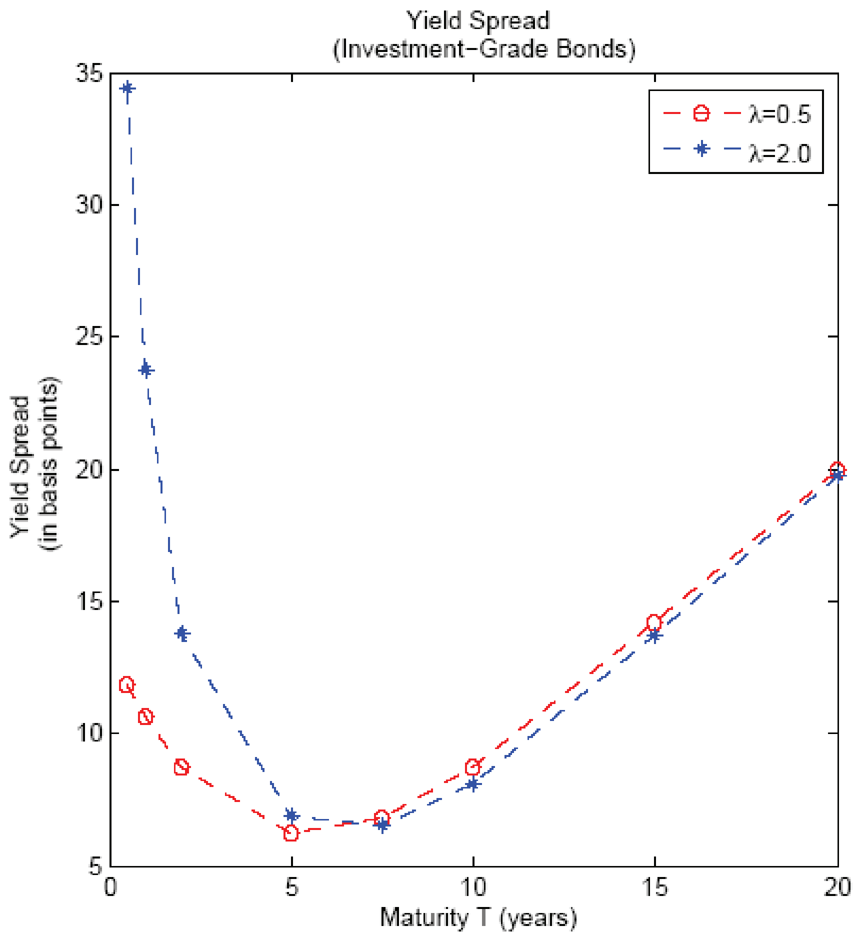

Figure 5: Yield spreads as a function of debt maturities with liquidity shock density $\lambda = 0.5, 2$ per year. The lines plot yield spreads (in basis points) of newly issued bonds as a function of maturity T ranging from 1 month to 20 years for firms with leverage ratio 20 percent. The dotted line represents yield spreads for $\lambda = 0.5$, and the star line represents yield spreads for $\lambda = 2$. The parameters chosen here are: $V = 100$, $r = 0.06$, $\delta = 0.07$, $\sigma_1 = 0.20$, $\alpha = 0.50$, $\tau = 0.35$ and $\overline{\zeta} = 0.9973$. The coupon rate c is exogenously given and equal to 0.075 per year for one dollar principal.

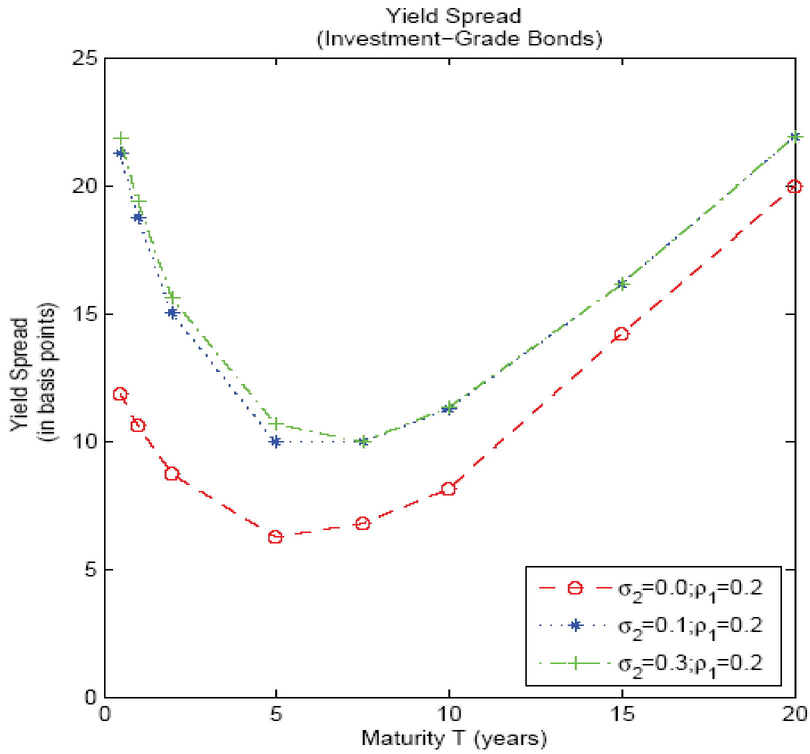

Figure 6: Yield spreads as a function of debt maturities for $\sigma_2 = 0, 0.1$ and 0.3. The lines plot yield spreads (in basis points) of newly issued bonds as a function of maturity T ranging from 1 month to 20 years for firms with leverage ratio 20 percent. The dotted line represents yield spreads for $\sigma_2 = 0$, the star line represents yield spreads for $\sigma_2 = 0.1$ and the cross line represents yield spreads for $\sigma_2 = 0.3$. The parameters chosen here are: $V = 100$, $r = 0.06$, $\delta = 0.07$, $\sigma_1 = 0.20$, $\alpha = 0.50$, $\tau = 0.35$, $\lambda = 0.50$, $\rho_1 = 0.2$, $\zeta_0 = 0.9973$ and $\zeta_m = 0.9973$. The coupon rate c is exogenously given and equal to 0.075 per year for one dollar principal.

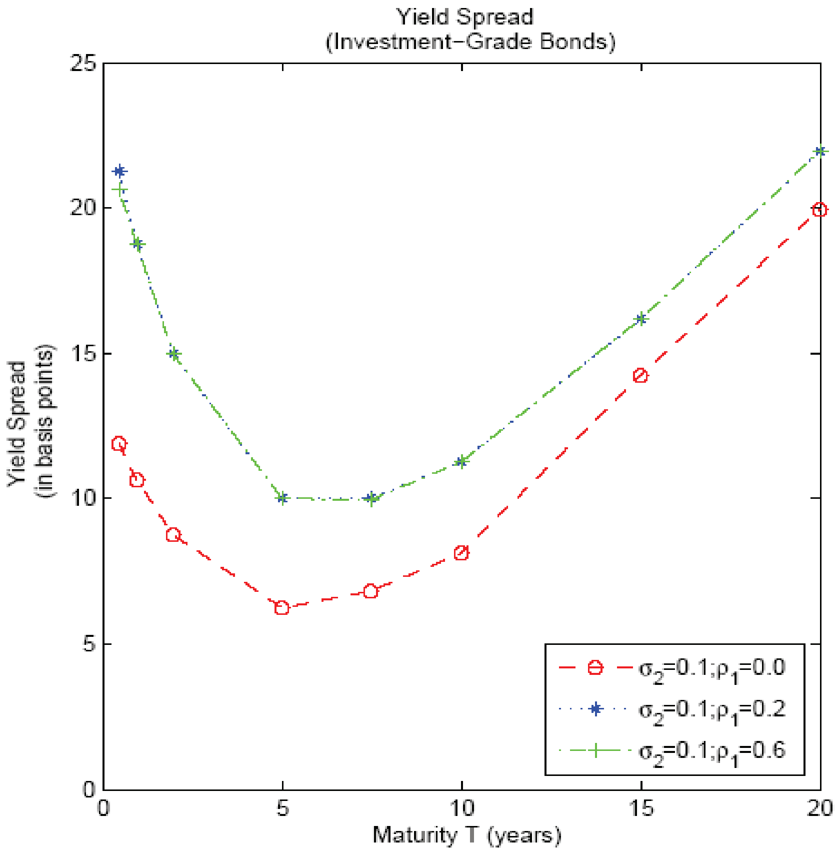

Figure 7: Yield spreads as a function of debt maturities for $\rho_1 = 0, 0.2$ and 0.6. The lines plot yield spreads (in basis points) of newly issued bonds as a function of maturity T ranging from 1 month to 20 years for firms with leverage ratio 20 percent. The dotted line represents yield spreads for $\rho_1 = 0$, the star line represents yield spreads for $\rho_1 = 0.2$ and the cross line represents yield spreads for $\rho_1 = 0.6$. The parameters chosen here are: $V = 100$, $r = 0.06$, $\delta = 0.07$, $\sigma_1 = 0.20$, $\alpha = 0.50$, $\tau = 0.35$, $\lambda = 0.50$, $\sigma_2 = 0.1$, $\zeta_0 = 0.9973$ and $\zeta_m = 0.9973$. The coupon rate c is exogenously given and equal to 0.075 per year for one dollar principal.

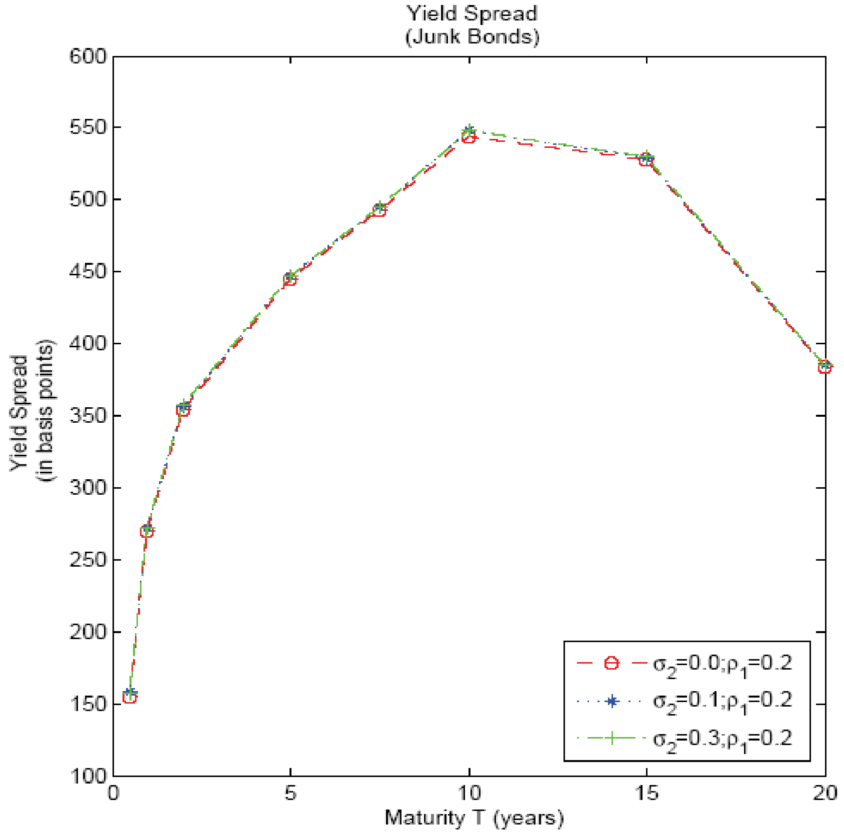

Figure 8: Yield spreads as a function of debt maturities for $\sigma_2 = 0, 0.1$ and 0.3. The lines plot yield spreads (in basis points) of newly issued bonds as a function of maturity T ranging from 1 month to 20 years for firms with leverage ratio 60 percent. The dotted line represents yield spreads for $\sigma_2 = 0$, the star line represents yield spreads for $\sigma_2 = 0.1$ and the cross line represents yield spreads for $\sigma_2 = 0.3$. The parameters chosen here are: $V = 100$, $r = 0.06$, $\delta = 0.07$, $\sigma_1 = 0.20$, $\alpha = 0.50$, $\tau = 0.35$, $\lambda = 0.50$, $\rho_1 = 0.2$, $\zeta_0 = 0.9973$ and $\zeta_m = 0.9973$. The coupon rate c is exogenously given and equal to 0.075 per year for one dollar principal.

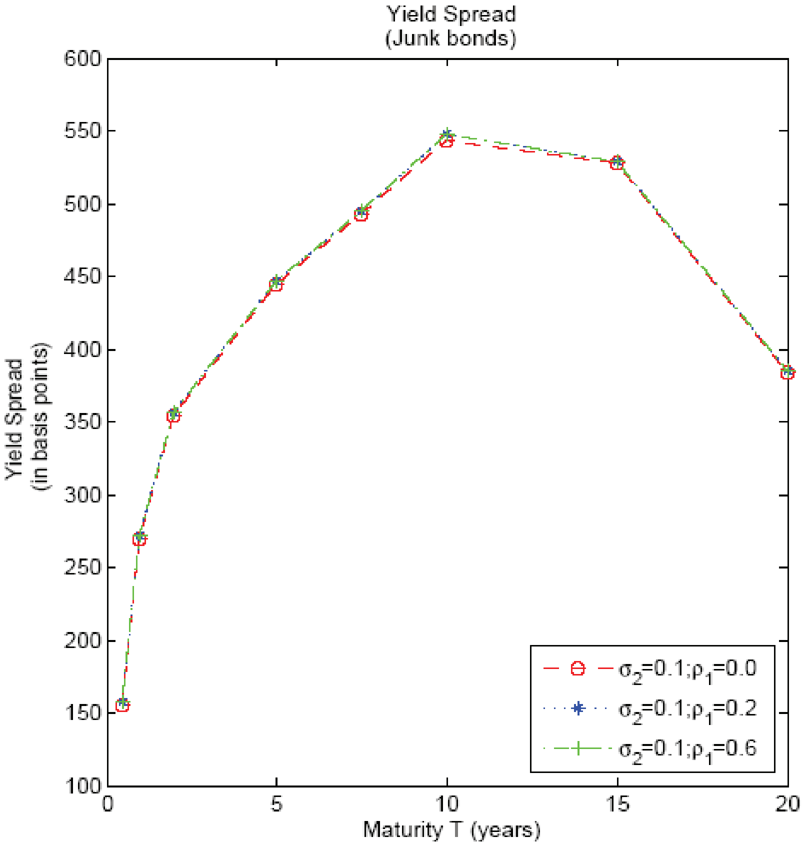

Figure 9: Yield spreads as a function of debt maturities for $\rho_1 = 0, 0.2$ and 0.6. The lines plot yield spreads (in basis points) of newly issued bonds as a function of maturity T ranging from 1 month to 20 years for firms with leverage ratio 60 percent. The dotted line represents yield spreads for $\rho_1 = 0$, the star line represents yield spreads for $\rho_1 = 0.2$ and the cross line represents yield spreads for $\rho_1 = 0.6$. The parameters chosen here are: $V = 100$, $r = 0.06$, $\delta = 0.07$, $\sigma_1 = 0.20$, $\alpha = 0.50$, $\tau = 0.35$, $\lambda = 0.50$, $\sigma_2 = 0.1$, $\zeta_0 = 0.9973$ and $\zeta_m = 0.9973$. The coupon rate c is exogenously given and equal to 0.075 per year for one dollar principal.

Table 1:

Characteristics of Optimally Leveraged Firms under Models with and without Liquidity Risk

This table compares the characteristics of optimally leveraged firms issuing debts with maturities ranging from 6 months to 20 years under models with and without liquidity risk. The model parameters take values as follows: V=100, r=0.06, δ=0.07, σ_1=0.20, α=0.50, τ=0.35, λ=0.50 and $\overline{\zeta}$=0.9973. The bankruptcy-triggering values are endogenously determined. The coupon rate c is exogenously given and set equal to 0.075 per year for one dollar principal. BV, DV, EV, FV, LR and YS are respectively abbreviations for Bankruptcy value, Debt value, Equity value, Firm value, Leverage Ratio and Yield Spread (in basis points).

	With Liquidity Risk							Without Liquidity Risk						
T	P	BV	DV	EV	FV	LR	YS	P	BV	DV	EV	FV	LR	YS
0.5	13.45	20.68	13.50	89.69	103.18	0.131	11.91	14.49	21.47	14.55	88.83	103.37	0.141	0
1	15.28	20.93	15.38	88.41	103.79	0.148	10.62	17.21	22.80	17.33	86.67	104.01	0.167	0
2	17.98	21.34	18.22	86.45	104.67	0.174	8.610	21.40	24.55	21.70	83.28	104.98	0.207	0.002
5	21.99	27.14	22.68	81.68	104.36	0.217	6.930	30.44	27.49	31.45	75.64	107.09	0.294	9.51
7.5	22.45	23.18	23.45	82.32	105.76	0.222	8.460	36.03	28.93	37.58	70.81	108.39	0.347	39.19
10	21.48	24.93	22.56	82.34	104.90	0.215	10.59	40.57	29.95	42.44	67.01	109.45	0.388	72.02
20	13.85	20.81	14.93	88.37	103.30	0.145	8.910	52.61	32.18	54.11	58.14	112.25	0.482	145.52

Table 2:
Characteristics of Optimally Leveraged Firms under Different Liquidity Shock Densities: λ =0.5, 2

This table compares the characteristics of optimally leveraged firms issuing debts with maturities ranging from 6 months to 20 years under different liquidity shock densities: λ=0.5, 2. Other model parameters take values as follows: V=100, r=0.06, δ=0.07, σ_1=0.20, α=0.50, τ=0.35 and $\overline{\zeta}$=0.9973. The bankruptcy-triggering values are endogenously determined. The coupon rate c is exogenously given and set equal to 0.075 per year for one dollar principal. BV, DV, EV, FV, LR and YS are respectively abbreviations for Bankruptcy value, Debt value, Equity value, Firm value, Leverage Ratio and Yield Spread (in basis points).

	λ=0.5							λ=2.0						
T	P	BV	DV	EV	FV	LR	YS	P	BV	DV	EV	FV	LR	YS
0.5	13.45	20.68	13.50	89.69	103.18	0.131	11.91	10.72	19.23	10.75	91.75	102.50	0.105	34.20
1	15.28	20.93	15.38	88.41	103.79	0.148	10.62	11.22	19.55	11.28	91.33	102.61	0.110	23.60
2	17.98	21.34	18.22	86.45	104.67	0.174	8.610	12.97	20.65	13.14	89.88	103.02	0.128	13.80
5	21.99	27.14	22.68	81.68	104.36	0.217	6.930	17.21	22.71	17.75	86.28	104.03	0.171	6.280
7.5	22.45	23.18	23.45	82.32	105.76	0.222	8.460	18.33	23.16	19.13	85.18	104.31	0.183	5.630
10	21.48	24.93	22.56	82.34	104.90	0.215	10.59	18.32	23.39	19.27	84.97	104.24	0.185	6.660
20	13.85	20.81	14.93	88.37	103.30	0.145	8.910	11.91	19.96	12.84	89.94	102.78	0.125	6.470

Table 3:
Characteristics of Optimally Leveraged Firms under Different σ_2 and ρ_1

This table compares the characteristics of optimally leveraged firms issuing debts with maturities ranging from 6 months to 20 years under different combinations of σ_2 and ρ_1. Other model parameters take values as follows: V=100, r=0.06, δ=0.07, σ_1=0.2, α=0.5, τ=0.35, λ=0.5, ζ_0=0.9973 and ζ_m=0.9973. The bankruptcy-triggering values are endogenously determined. The coupon rate c is exogenously given and set equal to 0.075 per year for one dollar principal. BV, DV, EV, FV, LR and YS are respectively abbreviations for Bankruptcy value, Debt value, Equity value, Firm value, Leverage Ratio and Yield Spread (in basis points).

Panel A: $\sigma_2 = 0.1$; $\rho_1 = 0.2$

T	P	BV	DV	EV	FV	LR	YS
0.5	13.490	20.930	13.534	89.605	103.139	0.131	20.85
1	15.389	21.922	15.488	88.096	103.584	0.149	18.86
2	18.259	23.274	18.493	85.758	104.250	0.177	15.33
5	22.722	24.995	23.419	81.887	105.306	0.222	11.06
7.5	23.088	25.032	24.043	81.378	105.421	0.228	12.57
10	22.285	24.885	23.365	81.824	105.188	0.222	14.26
20	14.250	21.391	15.304	88.001	103.305	0.148	11.17

Panel B: $\sigma_2 = 0.1$; $\rho_1 = 0.6$

T	P	BV	DV	EV	FV	LR	YS
0.5	13.482	20.866	13.492	89.647	103.139	0.131	20.79
1	15.383	21.914	15.482	88.102	103.584	0.149	18.83
2	18.271	23.292	18.504	85.745	104.249	0.178	15.35
5	22.695	24.970	23.391	81.913	105.304	0.222	11.07
7.5	23.131	25.087	24.084	81.334	105.419	0.228	12.64
10	22.336	24.952	23.415	81.770	105.185	0.226	14.33
20	14.126	21.236	15.197	88.107	103.304	0.147	10.99

Panel C: $\sigma_2 = 0.3$; $\rho_1 = 0.2$

T	P	BV	DV	EV	FV	LR	YS
0.5	13.487	20.924	13.530	89.609	103.139	0.131	21.67
1	15.395	21.929	15.493	88.092	103.585	0.150	19.44
2	18.276	23.293	18.508	85.743	104.251	0.178	15.73
5	22.585	24.840	23.276	82.031	105.307	0.221	11.21
7.5	23.339	25.299	24.296	81.127	105.423	0.230	13.07
10	22.238	24.827	23.315	81.875	105.190	0.222	14.31
20	14.144	21.198	15.178	88.129	103.306	0.147	11.08

Table 4:
Characteristics of Firms with Different Leverage Ratios 20% and 60%

This table reports the characteristics of firms issuing debts with maturities ranging from 6 months to 20 years with two different leverage ratios 20% and 60%. The model parameters take values as follows: V=100, r=0.06, δ=0.07, σ_1=0.20, α=0.50, τ=0.35, λ=0.50 and $\overline{\zeta}$=0.9973. The bankruptcy-triggering values are endogenously determined. The coupon rate c is exogenously given and set equal to 0.075 per year for one dollar principal. BV, DV, EV, FV and YS are respectively abbreviations for Bankruptcy value, Debt value, Equity value, Firm value and Yield Spread (in basis points).

Panel A: Leverage Ratio = 20%

	With Liquidity Risk						Without Liquidity Risk					
T	P	BV	DV	EV	FV	YS	P	BV	DV	EV	FV	YS
0.5	20.379	31.614	20.448	81.796	102.244	11.875	20.474	30.324	20.550	82.200	102.75	0
1	20.493	29.236	20.629	82.511	103.140	10.625	20.616	27.318	20.676	83.069	103.84	0
2	20.565	26.225	20.842	83.332	104.174	8.7500	20.697	23.749	20.995	83.980	104.98	0
5	20.394	22.594	21.049	84.146	105.205	6.2500	20.562	18.572	21.260	85.041	106.30	0
7.5	20.153	21.907	21.060	84.242	105.302	6.8184	20.359	16.347	21.346	85.384	106.73	2.20
10	19.950	22.301	21.025	84.099	105.124	8.6967	20.154	14.878	21.391	85.565	106.96	4.79
20	19.686	29.596	20.550	82.200	102.750	19.950	19.510	11.933	21.449	85.795	107.24	16.55

Panel B: Leverage Ratio = 60%

	With Liquidity Risk						Without Liquidity Risk					
T	P	BV	DV	EV	FV	YS	P	BV	DV	EV	FV	YS
0.5	48.965	72.587	48.783	32.514	81.297	155.00	48.834	72.328	48.941	32.627	81.568	144.41
1	51.339	68.618	51.289	34.177	85.466	269.65	51.477	68.213	51.524	34.349	85.873	249.38
2	54.731	64.168	54.078	36.028	90.106	354.50	54.920	63.019	54.708	36.472	91.191	308.88
5	60.476	62.108	55.817	37.162	92.979	444.86	60.592	54.729	59.621	39.748	99.369	310.25
7.5	64.385	65.955	54.044	35.980	90.024	492.72	63.304	50.829	61.859	41.239	103.10	294.00
10	68.287	72.884	50.219	33.531	83.750	543.71	65.191	48.125	63.381	42.254	105.64	279.85
20	61.319	87.638	39.611	26.379	65.990	384.12	69.201	42.328	66.558	44.372	110.93	244.34

Chapter 2

Hedge fund alphas: do they reflect managerial skills or mere compensation for liquidity risk bearing?

[1]

[1] A revised version of this manuscript is accepted to publish in the Journal of Financial and Quantitative Finance (JFQA).

2.1 Introduction

Do hedge funds deliver superior performance rooted in their managers' skills? This question has attracted a lot of attention ever since the late 1940s when Alfred Winslow Jones, a sociologist turned fund manager, established an investment fund as a general partnership with several characteristics which now define hedge funds. He developed the notion of hedge fund as a market-neutral strategy, by which long positions in undervalued securities would be offset and partially funded by other short positions. This "hedged" position effectively leveraged investment capital and allowed large bets with limited private resources. The term "hedge fund" has since then been extended to a wide variety of funds that rely on short-selling, leverage and dynamic portfolio strategies to yield superior performance. Given that hedge funds typically charge quite a high performance fee, averaging 20% of annual their gross returns[2], it is important to determine whether the latter is typically justified in light of their risk-adjusted performance. Indeed, investors should be willing to pay such fees if they believe that hedge fund managers have the ability to offset these additional costs. Judging by the tremendous growth of the asset under management by hedge funds since the late 90s, one may be tempted to conclude that yes, hedge funds have outperformed and delivered "net" alphas exceeding those realized by mutual funds or more traditional actively managed portfolios. Indeed, just at the start of the financial crisis, the 2008 Hedge Fund Asset Flows & Trends Report by HedgeFund.net and Institutional Investor News estimated that the total hedge funds' industry assets had reached \$2.68 trillion. it must be pointed out that assessing hedge funds' effective performance and dissociating their market timing and selectivity skills from their pure risk bearing compensations remains quite a challenge for most investors and fund advisors. Was this trend and this spectacular growth in assets under management motivated by hedge funds alphas? This question is indisputably hard to assess for most institutional and private investors as well as for most hedge funds advisors. The true ability of hedge funds managers to generate pure alphas also remains an open debate in the academic literature[3]. Recently, leveraging on the Bayesian framework proposed by Avramov and Wermers (2006), Avramov et al. (2007) have investigated the performance of optimal portfolio strategies in hedge funds. These strategies exploit predictability in (i) managerial

[2]In the hedge fund industry, the management fee is calculated as a percentage of the net asset value of the fund at the time when the fee becomes payable. Management fees typically range from 1% to 4% per annum, with 2% being the standard figure. A hedge fund's performance fee is calculated as a percentage of the fund's profits, counting both unrealized profits and actual realized trading profits. Hedge funds typically charge 20% of gross returns as a performance fee, but the range is again wide.

[3]See Section 2.2 for a literature review on the assessment of managers skills in the hedge fund industry.

skills, (ii) fund risk loadings, and (iii) benchmark returns. By examining the ex-post out-of-sample performance of a large number of hedge funds portfolios in this setting, the authors conclude that there exist subgroups of hedge funds that deliver significant outperformance. Moreover, they also show that predictability in managerial skills is the dominant source of investment profitability, indicating that portfolio strategies incorporating predictability in managerial skills outperform others mainly because they can choose hedge funds managers with higher expected future skills.

In this article, we wish to extend the former study by examining the joint impact of predictability and of an important omitted risk factor, namely liquidity risk, on the assessment of the performance of hedge funds. Liquidity refers to both the time and costs associated with the transformation of a given position into cash and vice versa. Typically, continuous-time arbitrage or equilibrium asset pricing models ignore liquidity since the cost and time required to transfer financial wealth into cash is assumed to be nil and since trading is often ruled out by most equilibrium asset pricing models. Yet, in practice, financial crises (such as in Asia or in Russia during the nineties), the debacle of the LTCM hedge fund or the most recent Subprime credit crisis suggest that at times of tight credit and market conditions, liquidity can decline and even temporarily dry out. This leads investors to aggressively bid for the safest, that is, the most liquid securities, which raises their price relative to the one of their less liquid counterparts. If market liquidity evolves randomly, securities or portfolios that covary more with liquidity, should offer a lower liquidity risk premium. Liquidity risk has recently been acknowledged as an important systematic source of risk for equity investments in the finance literature (see, e.g., Pastor and Stambaugh (2003); Gibson and Mougeot (2004)). In the following study we wish to answer a very simple question, namely, could it be that part of the superior performance reported by hedge funds is in fact a “pure” compensation for the liquidity risk exposures these funds take by making trades and following strategies which bear more liquidity risk? Similarly, we conjecture that some hedge funds strategies that take long/short bets and place one of their legs in illiquid assets or more generally that trade in less liquid securities markets should be more prone to the liquidity risk factor omission bias when it comes to assessing their performance.

More precisely, we shall follow Avramov et al. (2007), in that we will evaluate hedge fund performance for portfolio strategies that explicitly incorporate predictability in managerial skills, in fund risk loadings and in benchmark returns. We will measure the performance of these portfolios by relying on the Hasanhodzic and Lo (2007) six factor linear performance evaluation model augmented by a liquidity risk factor. The latter factor is constructed based on the Pastor and Stambaugh (2003) liquidity measure and its derived systematic liquidity risk factor. The main out-of-sample results suggest that once we account for this omitted liquidity risk factor, the significance of a large number of hedge fund portfolios' alphas disappears or is vastly reduced even when predictability in managerial skills is considered. More precisely, the results show that, except for the non-equity oriented and market-neutral styles based hedge fund portfolios, the liquidity betas are significantly positive and economically relevant[4]. Furthermore, for 40% of the hedge funds style portfolios, after adjusting for the effect of liquidity risk , the alphas turn out to be insignificant. More precisely, this trend is observed for the Convertible Arbitrage, Fund of Funds, Global Macro and Long/Short Equity Hedge styles portfolios whose alphas indeed become insignificant once we account for the effect of liquidity risk, and whose outperformance over similar strategies that ignore managerial skills effectively disappears. A similar effect of liquidity risk on the alphas of most hedge fund portfolios in the Event Driven and Emerging Markets styles is observed although the alphas associated with some of these portfolios when accounting for predictability in managerial skills still remain significant but much smaller. The only hedge funds style portfolios that yield significantly superior performance even after accounting for liquidity risk are the Market Neutral and Multi-Strategy Funds. These empirical results are robust to: (i) the choice of an alternative performance evaluation model (The Fung and Hsieh (2004) seven-factor performance evaluation model), (ii) the choice of an alternative liquidity risk proxy derived from Amihud (2002) liquidity measure, (iii) the exclusion of the January effect, and (iv) the exclusion of the recent financial crises impact.

We believe that these empirical results have a wide range of theoretical and practical implications: First, they show that introducing predictability in managerial skills is not sufficient to generate a "pure" and economically significant alpha within most hedge funds investment styles. Second, they suggest that liquidity risk plays an important role within many hedge funds styles and that a large fraction of the hedge funds superior performance documented by previous performance models actually represents a mere compensation for their liquidity risk exposures. Third, they invite us to reconsider and deepen our understanding of the role played by a significant omitted risk factor, namely liquidity risk, in order to help investors and fund advisors to make "informed" judgments regarding these vehicles' potentially understated systematic risk exposures and overstated alphas.

The organization of this article is as follows: Section 2.2 provides a literature survey. Section 2.3 explains the theoretical framework used in this article to construct hedge funds

[4]See the Appendix in Section 2.7 for the category of hedge fund styles.

portfolios, to measure liquidity risk and finally to estimate the hedge fund portfolios' performance. Section 2.4 describes the data. Empirical results are analyzed in Section 2.5. Finally, Section 2.6 provides the main conclusions.

2.2 Literature review

The identification and the quantification of hedge funds' performance is a widely studied yet still unresolved research issue in finance.

With a large sample of hedge fund data from 1988-1995, Ackermann et al. (1999) find that annualized Jensen alphas are significantly positive for hedge funds and range from 6% to 8% per year and that hedge funds consistently outperform mutual funds. Brown et al. (1999) examine the performance of off-shore hedge funds over the period 1989 through 1995 by using an *annual* database that includes both live and defunct hedge funds, and find that all but one (short-sellers) of ten hedge fund styles provide positive risk-adjusted returns. Moreover, they do not find performance persistence using two-way winner-and-loser contingency tables. Relying on two different excess return measures, *alphas* and *appraisal ratios*[5], Agarwal and Naik (2000) study the persistence in the performance of hedge funds using a multi-period framework, and find maximum persistence at the quarterly horizon suggesting that the persistence of hedge fund managers' performance is short term in nature.

The articles above employ single-factor models to measure hedge funds' performance. Liang (1999) employs an eight-asset-class-factor model to estimate alphas for equally-weighted hedge fund indexes by using a stepwise regression procedure, and his results suggest that, on a risk-adjusted basis, most hedge fund groups earn positive unexplained returns, and some of those unexplained returns are statistically significant. Edwards and Caglayan (2001) show that about 25% of hedge funds earn positive excess returns (six-factor Jensen alphas) and that the magnitude of funds' excess returns differs markedly across investment styles. In those two papers, it is also shown that hedge funds that pay managers higher performance fees generate higher excess returns, which, when combined with the findings of positive excess returns, suggests that fund managers skills may be a partial explanation for the superior performance generated by hedge funds.

Recently, Kosowski, Naik, and Teo (2007) argue that top hedge fund performance can

[5]In Agarwal and Naik (2000), alpha is measured as the return of a hedge fund using a particular strategy minus the average return for all hedge funds following the same strategy, and appraisal ratio is defined as the alpha divided by the residual standard deviation resulting from CAPM, a regression of the hedge fund return on the average return of all hedge funds following that strategy.

not be explained by luck or sample variability, and that hedge fund performance persists at annual horizons. They develop a powerful bootstrap and Bayesian procedure which helps overcome the short-sample problem in hedge fund returns. With their estimates, sorts on Bayesian posterior fund alphas result in a 5.5%/year increase in the alpha spread between the top and bottom decile hedge funds. In a paper that motivated the current study, Avramov et al. (2007) evaluate hedge fund investment strategies when returns are predictable. They show that the strategies that incorporate predictability in managerial skills deliver significantly higher alphas (3-5%). Their findings suggest that (i) some managers possess superior skills, and (ii) that incorporating predictability in managerial skills allows investors to identify those well performing managers.

However, Malkiel and Saha (2005) show that the practice of voluntary reporting and the backfilling of only favorable past returns can cause returns calculated from hedge fund databases to be biased upwards. Moreover, the considerable attrition rate in the hedge funds' industry results in substantial survivorship bias in the returns of indexes composed of only currently operating funds. After adjusting for such biases, they find that hedge funds significantly underperform, on average, their benchmarks. Getmansky, Lo, and Makarov (2004) argue that the most likely sources of serial correlation in hedge fund returns are illiquidity exposure and return smoothing. They propose an econometric model of return smoothing and develop estimators for the smoothing profile as well as a smoothing adjusted Sharpe ratio. Using the return smoothing model, Getmansky, Lo, and Makarov (2004) demonstrate that the performance persistence at quarterly horizons in hedge funds documented by Agarwal and Naik (2000) and other papers can be simply traced to illiquidity-induced serial correlation in hedge fund returns. Aragon (2007) uses hedge fund share restrictions like lockup provisions and redemption notice periods as transaction costs approximation. He shows that the relation between share restrictions and hedge fund returns is positive and that the alphas of hedge fund returns will disappear when such transaction costs are considered. As in Amihud and Mendelson (1986), Aragon (2007) also reports the existence of a concave relation between share restrictions and hedge fund returns, which would suggest that long-horizon investors hold hedge funds with longer lockup provisions and redemption notice periods and thus require higher returns on those funds.

As for liquidity risk which has been introduced into the hedge fund performance evaluation in this article, its role in asset pricing has been intensively discussed in the finance literature. It is worth mentioning that liquidity risk as considered in this article is distinct from the concepts of liquidity examined in Getmansky, Lo, and Makarov (2004) and in

Aragon (2007). Indeed, we focus on liquidity risk stemming from the fact that certain assets held by hedge funds may covary with a systematic liquidity risk proxy. In contrast, the previous authors focus primarily on illiquidity as a cost factor which induces serial correlation in hedge fund returns and which may require higher expected returns as well.

Most of the market microstructure studies that investigate the relation between liquidity and asset returns focus on the level of liquidity as a trading characteristic of a financial asset, and argue that investors holding illiquid assets are compensated for bearing this cost through higher future expected returns. A more recent strand of the market microstructure literature pioneered by Brennan and Subrahmanyam (1996) and later by Chordia, Roll, and Subrahmanyam (2000) has also emphasized that there is commonality driving stock markets' liquidity and thus a systematic component to their liquidity cross-sectional variations. A somewhat related strand of literature begins with studies showing that firm specific liquidity fluctuates over time and that a market-wide component generates these liquidity fluctuations. Pastor and Stambaugh (2003) show that systematic liquidity is a priced risk factor. They develop a measure of aggregate liquidity based on daily price reversals and show that assets whose returns covary highly with this aggregate liquidity measure earn higher expected returns (7.5% after adjusting for exposures for the market return as well as size, value, and momentum factors) than do assets whose returns exhibit low covariation with aggregate liquidity. Acharya and Pedersen (2005) explicitly solve a simple equilibrium model with liquidity risk. In their liquidity-adjusted capital asset pricing model, a security's required return depends on its expected illiquidity as well as on the covariances of its own return and liquidity with the market return and liquidity. Using the Amihud (2002) illiquidity measure defined as the ratio of the daily absolute return to trading volume (in millions of US dollars) for the day, they estimate the combined effect of illiquidity and liquidity risk to be approximately 4.6% per year, with 1.1% due to the effect of pure liquidity risk. Recently, Sadka (2006) successfully shows how the liquidity risk premium can explain the momentum and post-earnings-announcement drift (PEAD) anomalies in portfolio returns[6]. Billio et al. (2007) argue that liquidity risk is potentially a common risk factor affecting certain hedge fund strategies in the down-state of the market, when volatility is high and returns are very low since they find that in the high-volatility regime (when the market is trending down) most of the strategies are negatively and significantly exposed to their Large-Small risk

[6]See, e.g., Amihud and Mendelson (1986), Constantinides (1986), Brennan and Subrahmanyam (1996), Brennan, Roll, and Subrahmanyam (1996), Amihud and Mendelson (2001), Huberman and Halka (2001), Amihud (2002), Chordia et al. (2002), Huang (2003), Ljungqvist and Richardson (2003), Pastor and Stambaugh (2003), Gibson and Mougeot (2004), Acharya and Pedersen (2005), and Sadka (2006) for more discussions of liquidity measures and liquidity risk.

factor.

A recent paper by Sadka (2009) is closely related to our paper in that it also examines whether liquidity risk is priced in hedge funds. The results in Sadka(2009) suggest that portfolios composed of hedge funds with larger loadings on liquidity risk factor earn higher returns and that the return of the high-minus-low liquidity loading portfolio is significant and around 6% annually. The main difference between our paper and Sadka (2009) is that relying on portfolios formed by incorporating predictability in managerial skills rather than on portfolios sorted by liquidity risk loadings, we not only investigate how liquidity risk affects hedge fund performance, but also study whether predictability in managerial skills is really effective in generating higher excess returns even after accounting for liquidity risk.

2.3 The hedge funds portfolio allocation model

In this article, the approach used to form optimal hedge funds portfolios follows Avramov et al. (2007). Following the latter study, we assume that managerial skills exist in the hedge fund industry and are predictable. Relying on predictability can help investors to select hedge fund managers with superior skills. Therefore, within such a portfolio allocation model, we can investigate whether predictability in managerial skills is really effective in so far as it allows one to incorporate highly skilled hedge fund managers in portfolios that subsequently generate higher excess returns, even after accounting for liquidity risk.

Assume that there are several types of Bayesian optimizing investors who differ from each other with respect to their beliefs about the possibility for hedge fund managers to possess asset selection skills and benchmark timing abilities. To be precise, these types of investors differ with respect to their views about the parameters governing the following hedge fund return generating model

$$\begin{aligned} r_{i,t} &= \alpha_{i,0} + \alpha_{i,1}^{'} z_{t-1} + \beta_{i,0}^{'} f_t + \beta_{i,1}^{'} \left(f_t \otimes z_{t-1} \right) + \epsilon_{i,t}, \\ f_t &= a_f + A_f z_{t-1} + \epsilon_{f,t}, \\ z_t &= a_z + A_z z_{t-1} + \epsilon_{z,t}, \end{aligned} \tag{2.1}$$

where $r_{i,t}$ is the return of hedge fund i in excess of riskless rate in month t. z_t is a vector of M business cycle variables observed at the end of month t, f_t is a vector of K zero-cost benchmarks, $\beta_{i,0}(\beta_{i,1})$ is the fixed (variable) component of fund risk loadings and $\epsilon_{i,t}$ is fund-specific event, which is assumed to be uncorrelated across hedge funds and over time, and normally distributed with mean zero and variance ψ_i. Business cycle variables z_t are modeled by a vector autoregression of order one.

Hedge fund managerial skills are captured by $\alpha_{i,0} + \alpha^{'}_{i,1} z_{t-1}$, which is composed of the fixed component $\alpha_{i,0}$ and the predictable component $\alpha^{'}_{i,1} z_{t-1}$. Note that the predictability of managerial skills is explained by public information instead of the private information possessed by hedge fund managers. This statement is consistent with the fact that the private information of the manager is correlated with the selected public information based business cycle variables.

Overall, the model for hedge fund returns described by Eq. (2.1) captures the potential predictability in managerial skills ($\alpha_{i,1} \neq 0$), in fund risk loadings ($\beta_{i,1} \neq 0$) and in benchmark returns ($A_f \neq 0$).

Following Avramov and al. (2007), we consider three specific types of hypothetical Bayesian investors, who have different views concerning the existence of hedge fund managerial skills in timing the benchmarks and selecting financial securities, and compare the performance of long-only constrained portfolio strategies for these three types of investors.

Forming hedge fund portfolio strategies based on such a methodology allows us to study the economic significance of fund return predictability and in particular of managerial skills predictability.

2.3.1 The dogmatist

The first type of investor considered is the dogmatist. This name is chosen to reflect the fact that this investor has extreme prior beliefs about the potential for managerial skills. Indeed, it is assumed that the dogmatist rules out the possibility for superior managerial skills. In this case, $\alpha_{i,0}$ is fixed at -$\frac{1}{12}$expense where *expense* is the hedge fund's annual reported expense ratio[7] and $\alpha_{i,1}$ is equal to zero. Thus, the dogmatic investor does not believe that hedge fund allocation decisions can be improved through seeking managers with private skills.

The dogmatist is further divided into three sub-types based on their beliefs about the predictability of fund risk loadings and benchmark returns. The first sub-type rules out any possible predictability, and we call him the no-predictability dogmatist (ND). The second one (PD-1) is a predictability dogmatist, but he just believes in the predictability of fund risk loadings ($\beta_{i,1} \neq 0$). The third (PD-2) believes in the predictability of both fund risk loadings

[7] To our knowledge, the data on the fund's annual reported expense ratio is not available, we therefore set it to be 2% (the same hereinafter). Setting other values for *expense* does not change our results. In Avramov and Wermers (2006), $\alpha_{i,0}$ is equal to -$\frac{1}{12}$(expense+turnover) where *turnover* is the fund's annual turnover which is even more difficult to obtain in the case of hedge funds.

and benchmark returns ($\beta_{i,1} \neq 0$ and $A_f \neq 0$). For both PD-1 and PD-2, the predictability in hedge fund returns based on public information can be exploited to improve hedge fund allocation decisions.

2.3.2 The agnostic

The second type of investor is agnostic with completely diffuse prior beliefs about the existence and the level of managerial skills. Particularly, the skill level $\alpha_{i,0} + \alpha_{i,1}^{'} z_{t-1}$ is assumed to have mean -$\frac{1}{12}$expense and unbounded variance. For the agnostic, prior beliefs are non-informative and hedge fund manager skills are completely determined by the data.

Unlike Avramov and Wermers (2006) and Avramov et al. (2007), we assume that there are only two types of agnostic investors instead of five. The first investor (PA-1) believes that only managerial skills are predictable, while the second one (PA-2) believes that managerial skills, factor loadings and benchmark returns are all predictable. The two types of investors respectively correspond to the investors PA-3 and PA-4 in Avramov and Wermers (2006) and Avramov et al. (2007).

We did not consider three other types of agnostic investors for whom the predictability in managerial skills is excluded. As shown in Avramov and Wermers (2006) and Avramov et al (2007), the predictability in managerial skills is the main source of outperformance. The investors who admit the existence of managerial skills, but deny its predictability, do not outperform other investors who rules out managerial skills. Since the goal of this study is to examine whether exploiting business cycle variables that can potentially predict the private skills of hedge fund managers can still improve investment performance after accounting for liquidity risk, we decided not to consider those other types of agnostic investors. The same arguments apply to the skeptic investor types who will be discussed in the next subsection.

2.3.3 The skeptic

The skeptic is an investor who believes in the existence of managerial skills, but his beliefs are bounded. Like in the previous subsection, we consider two types of investors. One (PS-1) believes that managerial skills are predictable, while the other (PS-2) believes that hedge fund allocation decisions can be improved by exploiting the business cycle variables that potentially forecast fund risk loadings, benchmark returns and managerial skills.

When skills may vary over time, an investor's prior belief can be modeled, like in Kandel and Stambaugh (1996), as if the investor observed a hypothetical sample of T_0 months in which there are no managerial skills based on either private or public information, and the mean and variance of fund returns, benchmark returns and business cycle variables in the hypothetical sample are equal to those in the actual sample.

Formally, for the skeptic, the prior mean of $\alpha_{i,1}$ is zero and the prior mean of $\alpha_{i,0}$ equals -$\frac{1}{12}$expense. The joint prior standard errors of these parameters depend on T_0. The choice of T_0 is determined by the following equation:

$$T_0 = \frac{s^2}{\sigma_\alpha^2}\left(1 + M + SR_{max}^2\right), \tag{2.2}$$

where SR_{max}^2 is the largest attainable Sharpe ratio based on investments in the benchmarks only, M is the number of business cycle variables and s^2 is the cross-section average of the sample variance of the residuals in Eq. (2.1) (the first equation). The proof can be found in the Appendix of Avramov and Wermers (2006).

2.3.4 Optimal portfolios formation

At each time t, there are N_t hedge funds defining the investment opportunity set, with N_t varying over time. We follow Avramov et al. (2007) in that each investor forms his portfolio by maximizing the conditional expected value of a quadratic utility function

$$U\left(W_t, R_{p,t+1}, a_t, b_t\right) = a_t + W_t R_{p,t+1} - \frac{b_t}{2} W_t^2 R_{p,t+1}^2, \tag{2.3}$$

where W_t denotes the time t invested wealth, b_t reflects the absolute risk aversion parameter and $R_{p,t+1}$ is the realized excess return on the optimal of hedge funds computed as $R_{p,t+1} = 1+r_{ft}+w_t^{'}r_{t+1}$, with r_{ft} being the risk-free interest rate, r_{t+1} denoting the vector of excess fund returns and w_t denoting the vector of optimal hedge fund allocations. Taking conditional expectations on both sides of Eq. (2.3) yields the following optimization problem

$$w_t^* = argmax_{w_t \geq 0}\left\{w_t^{'}\mu_t - \frac{1}{2\left(1/\gamma_t - r_{ft}\right)} w_t^{'}\Lambda_t^{-1} w_t\right\}, \tag{2.4}$$

where $\gamma_t = \left(b_t W_t\right) / \left(1 - b_t W_t\right)$ is the relative risk-aversion parameter, $\Lambda_t = \left[\Sigma_t + \mu_t \mu_t'\right]^{-1}$, with μ_t and Σ_t being respectively mean vector and variance matrix of future hedge fund returns. The possibility of leveraging and short selling is excluded when forming optimal hedge funds' portfolios.

Investors update their prior beliefs as soon as they obtain new information and the posterior densities of the parameters are obtained by combining the likelihood functions and the prior distributions. With such densities, investors can calculate the Bayesian predictive distribution of hedge fund returns (vector) r_{t+1}, from which the mean vector μ_t and variance matrix Σ_t of hedge fund excess returns are derived.

For each of the seven types of investors, we use the excess return on the value-weighted S&P500 index as the benchmark factor to derive his optimal portfolio. Other benchmark specifications with more factors can be considered, however, the limitation due to a limited hedge fund data sample makes it less convenient to use those multi-factor specifications. In addition, using the excess return on the value-weighted S&P500 index as the benchmark factor allows us to compare the results with the ones obtained by Avramov and al (2007).

2.3.5 Performance evaluation model

We use the Hasanhodzic and Lo (2007) six-factor linear model and its extension to evaluate the hedge funds portfolios' performance[8]. This choice is motivated by the fact that this model accounts for a broad cross-section of meaningful risk exposures for the typical hedge funds (including equity, interest rate, currencies, commodities, credit and volatility risk factors), due to their broad investment mandates and that it is widely accepted in the recent hedge funds literature. The six risk factors are: (1) the excess return on the S&P500 Index; (2) the excess return on the US Dollar Index; (3) the excess return on the Goldman Sachs Commodity Index; (4) the excess return on the Lehman Corporate AA Intermediate Bond Index; (5) the return spread on the Lehman BAA Corporate Bond Index and the Lehman Treasury Index, and (6) the first-difference of the month-end value of the Chicago Board Options Exchange (CBOE) Volatility Index.

We regress each portfolio's monthly returns in excess of the one-month T-bill rate on the six factors

$$\begin{aligned} r_{i,t} = {} & \alpha_i + \beta_{i,1} SP500_t + \beta_{i,2} USDX_t + \beta_{i,3} GSCI_t + \beta_{i,4} LHCAAI_t \\ & + \beta_{i,5} SPREAD_t + \beta_{i,6} VIX_t + \upsilon_{i,t} \end{aligned} \tag{2.5}$$

This performance evaluation model is different from the Fung and Hsieh (2004) seven-factor model used in Avramov et al. (2007) which relies on the so-called primitive trend following strategy (PTFS) risk factors to capture the well-documented option-like payoffs in hedge fund returns. We are going to use the Fung and Hsieh (2004) model to evaluate the performance of optimal hedge fund portfolios in the section of *'Robustness tests'*.

[8]Hasanhodzic and Lo (2007) use the linear model in order to replicate hedge fund returns rather than to evaluate performance.

2.3.6 Liquidity risk factors

In what follows, we will discuss the construction of the liquidity risk factor which is added to the performance model described in the previous section in order to examine whether a liquidity risk factor can explain the previously documented abnormal performance of hedge funds even after accounting for the existence of predictable managerial skills. The liquidity risk factor is constructed by using the liquidity measure proposed by Pastor and Stambaugh (2003). In that paper, the illiquidity of stock i in year y is defined as the ordinary least squares estimate of $\gamma_{i,t}$ in the following regression

$$r^e_{i,d+1,y} = \theta_{i,y} + \phi_{i,y} r_{i,d,y} + \gamma_{i,y} sign\left(r^e_{i,d,y}\right) v_{i,d,y} + \epsilon_{i,d+1,y}, \tag{2.6}$$

where $r_{i,d,y}$ is the return on stock i on day d in year y; $r^e_{i,d,y} = r_{i,d,y} - r_{m,d,y}$, $r_{m,d,y}$ being the return on the value-weighted CRSP index on day d in year y; and $v_{i,d,y}$ is the trading volume (in millions of US dollars) for stock i on day d in year y.

This liquidity measure ($\gamma_{i,t}$) focuses on an aspect of illiquidity associated with temporary price fluctuations induced by the order flow. The idea behind this measure is that, if signed volume is viewed roughly as a proxy for the *order flow*, then greater illiquidity is reflected by a greater tendency for the order flow in a given direction on day d to be followed by a price change in the opposite direction on day $d + 1$. Essentially, greater illiquidity corresponds to stronger volume-related return reversals, and in this respect this measure follows the same line of reasoning as the model and the empirical evidence presented by Campbell, Grossman, and Wang (1993). They find that returns accompanied by high volume tend to be reversed more strongly, and they explain how this result is consistent with a model in which some investors are compensated for accommodating the liquidity demands of others. $\gamma_{i,y}$ is generally negative and larger in absolute level when liquidity is lower.

Stocks are separated into 25 portfolios $p = 1, 2, ..., 25$ according to their annual illiquidity measures at the end of the previous year. The first portfolio is composed of the most liquid stocks, while the least liquid stocks are in the last portfolio. For each portfolio p, we calculate its return in month t as

$$r^p_t = \sum_{i\ in\ p} w^{ip}_t r^i_t, \tag{2.7}$$

where w^{ip}_t are either equal or value-based weights, depending on the specification. In the following, we only present the empirical results estimated from equally-weighted returns, the results estimated from value-weighted returns are quantitatively similar.

The liquidity risk factor, denoted by $LIQRISK^{ps}$, is then defined as the return on the least liquid portfolio minus the return on the most liquid portfolio, and its value in month t can be written as

$$LIQRISK_t^{ps} = r_t^{25} - r_t^1 \tag{2.8}$$

By definition, this factor can be interpreted as the return that investors are willing to give up for holding more liquid stocks.

The liquidity risk factor $LIQRISK^{ps}$ can then be incorporated into Eq. (2.5) to study the effect of liquidity risk on the performance of the optimal hedge funds portfolios formed according to the portfolio optimization scheme described in Section 2.3.4.

In order to check the robustness of the results, we also measure the performance of the optimal hedge funds portfolios using another liquidity risk factor, denoted by $LIQRISK^{amh}$, namely based on the liquidity risk measure proposed by Amihud (2002). The illiquidity of stock i is now defined as the ratio of its daily absolute return to the daily trading volume (in millions of US dollars). Specifically, this measure equals $|R_{iyd}| / VOL_{iyd}$ where R_{iyd} is the return on stock i on day d of year y and VOL_{iyd} is the respective daily trading volume. This ratio gives the absolute (percentage) price change per dollar of daily, or the price impact of, the order flow. The intuition behind this illiquidity measure is as follows: a stock is less liquid if its price moves substantially in response to a given trading volume. This follows Kyle's concept of illiquidity as the response of price to order flow and Silber's (1975) measure of thinness, defined as the ratio of absolute price change to absolute excess demand for trading.

The annual average illiquidity of stock i over year y is equal to the sum of the daily illiquidities during this year divided by the number of available trading days, and it can be described as

$$ILLIQ_{iy} = \frac{1}{D_{iy}} \sum_{t=1}^{D_{iy}} |R_{iyd}| / VOL_{iyd}, \tag{2.9}$$

where D_{iy} is the number of days for which data are available for stock i in year y. Amihud (2002) shows empirically that $ILLIQ$ is positively related to measures of price impact and fixed trading costs, suggesting that the illiquidity measure $ILLIQ$ may be interpreted as trading costs which include broker fees, bid-ask spread, market impact, and search costs.

As before, we estimate the illiquidity of each stock and then divide stocks into 25 portfolios according to their absolute illiquidity measures at the end of the previous year. The Amihud liquidity risk factor $LIQRISK^{amh}$ is defined as the return on the least liquid portfolio minus the return on the most liquid portfolio.

In this article, we only use equity-based liquidity risk factors, which strictly speaking should only be relevant for assessing the performance of hedge funds strategies operating within this asset class. This choice is motivated by the difficulty to collect analogous data to construct the Pastor and Stambaugh or the Amihud liquidity risk factors for other asset classes. We realize that this empirical choice is sub-optimal for non-equity based hedge funds strategies but it can be partially justified to the extend that liquidity risk tends to co-vary across different securities markets. In the concluding section, we mention that, as an extension of this study, it would be worth exploring whether liquidity risk factors constructed from the direct primary markets price and volume data in which the hedge funds operate, could help us better explain and disentangle the performance of these non-equity hedge funds portfolios.

2.4 Data

The hedge fund data used in this paper is taken from the Lipper TASS database which has been intensively used in academic studies. The database is divided into two parts: *Live* and *Graveyard* hedge funds. Although the TASS database dates back to February 1977, we start our study in January 1994 when TASS started to report data for *Graveyard* hedge funds in order to avoid the well documented survivorship bias. Fung and Hsieh (2000) estimate the magnitude of the survivorship bias to be around 3% per year, and Liang's (2000) estimator is 2.24% per year[9]. The study thus extends from January 1994 to December 2006.

Another well known bias associated with hedge fund returns is the backfilling bias[10]. This bias is due to the fact that hedge funds may choose when and whether or not to backfill reported returns data to vendors. It is also likely that hedge funds with good historical return data may go on to seek outside investors and report their returns to database vendors while funds with poor records do not reach this stage. This bias likely shifts hedge fund returns upwards. In order to mitigate the impact of the backfilling bias, we will not use the first 12-month return data for each hedge fund.

In addition, hedge funds have been droped in the case that they: (i) did not report net-of-fees returns, (ii) reported returns in other currencies than the U.S dollars, (iii) reported returns less frequently than monthly, and (iv) had less than 24 monthly returns.

After adjusting for the biases and deleting some funds that do not satisfy the above

[9]Refer to Schneewies and Spurgin (1996), Brown et al. (1997), Fung and Hsieh (1997), Carpenter and Lynch (1999), Horst et al. (2001), and Baquero (2005) for more studies attempting to quantify the degree and impact of survivorship bias in the hedge fund industry.

[10]See Posthuma and van der sluis (2003).

requirements, we have a total of 4698 hedge funds, of which 2743 are *Live* funds, in our sample. Table 1 presents some summary statistics for the entire sample. The Event Driven, Long/Short Equity Hedge and Multi-Strategy hedge funds yielded the highest average annualized returns and Sharpe ratios (except for Long/Short Equity Hedge) during the period from January 1994 to December 2006, this can perhaps explain the phenomenon that the biggest inflows in US-dollar terms were experienced by these three types of hedge funds in the first quarter of 2007[11]. In contrast, Dedicated Short Bias performed rather poorly during the same period, with negative average annualized return and Sharpe ratios. Comparing Table 1 with Table 2 which *only* shows summary statistics for the *Live* hedge funds during the same period, we find that average annualized returns and Sharpe ratios for all hedge fund styles but Dedicated Short Bias are higher in the case of *Live* hedge funds, consistent with the previous findings by Fung and Hsieh (2000) and Liang (2000).

One feature that is apparent from Table 2 is the positive average lag-one autocorrelation of hedge fund returns. The autocorrelations documented for most of the hedge fund styles, specially for Convertible Arbitrage, Event Driven, Emerging Markets, Fixed Income Arbitrage, Fund of Funds and Multi-strategy, are significantly positive and high. We will account for this hedge funds return characteristic subsequently when we conduct robustness checks on the performance results.

Daily return and volume data for other financial instruments are downloaded from CRSP. In order to construct the liquidity risk factors, $LIQRISK^{ps}$ and $LIQRISK^{amh}$, we use common stocks traded on the New York Stock Exchange (NYSE) and on the American Stock Exchange (AMEX). The stocks traded on NASDAQ are excluded since their volume data include inter-dealer data which result from a different trading mechanism. A stock will be considered in year y only if it satisfies the following criteria: (i) the stock price at the end of year $y-1$ lies between \$5 and \$1000, (ii) the stock must be listed at the end of year $y-1$, and (iii) the stock has return and volume data for more than 100 days during year $y-1$.

Figure 1 plots two monthly aggregate liquidity series obtained by taking the average of the individual-stock Pastor and Stambaugh (2003) and Amihud (2002) liquidity measures, and then multiplying by m_t/m_1, where m_t is the total dollar value at the end of month $t-1$ of the stocks included in the average in month t and month 1 corresponds to August 1962. The multiplier m_t/m_1 reflects the cost of a trade whose size is commensurate with the overall size of the stock market. It is not difficult to observe that liquidity is usually significantly lower and the return for liquidity thus significantly higher during the periods characterized

[11]See the Lipper TASS Asset Flows Report, first quarter 2007.

by liquidity crises such as: 10/1987 (stock market crash), 6-10/1998 (Russian default and LTCM crisis) and 2000-2001 (dot.com bubble crash).

Table 3 presents the correlation matrix of Hasanhodzic and Lo (2007) benchmark factors and the liquidity risk factors $LIQRISK^{ps}$ and $LIQRISK^{amh}$. We observe that the correlations between the market risk factor S&P500 and both liquidity risk factors are significantly negative: -0.609 and -0.527, implying that investors require higher returns for less liquid securities when the market is trending down. This result is consistent with the concept of "Flight to Quality" during market downturns. Also, the significant negative correlation between the market risk factor and the volatility factor (-0.703) suggests that volatility increases when the market goes down. Finally, the correlation between the two liquidity risk factors is very high: 0.892, thus Pastor and Stambaugh (2003) and Amihud (2002) liquidity risk measures seem to capture a similar trend in liquidity risk over time.

2.5 Empirical results

This section presents the out-of-sample performance results for the hedge funds' portfolio strategies that are optimal from the perspective of the seven types of investors described in Section 2.3. At the end of each year from 1995 to 2005, the portfolio for each investor is formed by using the previous 24-month information. Due to the fact that lockup provisions and redemption notice periods are common in the hedge funds industry, it takes time for investors to withdraw money from hedge funds, and we therefore reform portfolios only once every 12 months[12].

We follow Avramov et al.'s (2007) selection of the four business cycle instrumental variables, namely the treasury yield; the default spread defined as the yield difference between Moody's Baa rated and Aaa rated bonds; the term spread defined as the yield difference between Treasury bonds with more than 10 years to maturity and the 3 months T-Bill rate and the contemporaneous monthly Chicago Board Options Exchange (CBOE) Volatility Index. In the finance literature, the first three variables are often used to predict stock returns. To the extend that many hedge funds engage in volatility bets, the fourth variable should allow investors to predict hedge fund managerial skills over the stock market volatility cycle.

[12] Avramov et al. (2007) argue that reforming portfolios over shorter horizon does not basically change the results.

2.5.1 Descriptive statistics

Table 4 presents summary statistics for the seven optimal portfolio returns reported by hedge fund styles, including mean, minimum, maximum, standard deviation, annualized Sharpe ratios (SR), skewness and kurtosis. These hedge fund style statistics are not reported for the Dedicated Short Bias style because there were too few hedge funds available for this particular style.

Table 4 shows that the optimal portfolio strategies, which incorporate predictability in managers' skills into their investment decisions, earn higher average returns than the strategies that exclude manager skills, though the difference is small for Long/Short Equity Hedge and Managed Futures funds. For example, for Emerging Markets funds, the average return generated by the strategy PS-1 is 2.1%/month, 1.0%/month higher than the highest average return generated by the dogmatic investors. It is worth noting that, for Multi-Strategy funds, the average returns generated by all the strategies PS-1, PS-2, PA-1 and PA-2 are almost 100% higher than those generated by the strategies ND, PD-1 and PD-2.

Looking at the Sharpe ratios, the results are however mixed. For portfolios representative of the Fund of Funds, Global Macro, Long/Short Equity Hedge and Managed Futures categories, the strategies ND, PD-1 and PD-2 generate higher Sharpe ratios. For funds in Convertible Arbitrage, Equity Market Neutral and Multi-Strategy, the highest Sharpe ratios generated by the strategies which do and do not incorporate predictable manager skills are quite close.

The combination of higher average returns and relatively lower Sharpe ratios implies that the returns generated by the strategies PS-1, PS-2, PA-1 and PA-2 are more volatile, which is also apparent from Table 4. One potential explanation of the more volatile returns generated by the strategies PS-1, PS-2, PA-1 and PA-2 may be due to the fact that the number of funds in which these strategies invest is much lower over time[13], indicating that these strategies are less diversified and thus more volatile.

In addition, the returns for many hedge fund styles portfolios, such as for Event Driven, Emerging Markets, Convertible Arbitrage, display negative skewness and are left tailed, implying that these portfolios may suffer from infrequent but extreme losses.

[13]See Table 5 in the following.

2.5.2 Analysis of the optimal portfolios' components

Before evaluating the out-of-sample performance of the optimal portfolios, we analyze some characteristics of their hedge fund components in order to identify potential patterns characterizing these optimal hedge fund strategies. The results are shown in Table 5.

The portfolio characteristics considered here are: the number of hedge funds, the age of the hedge funds since their inception (in years), the redemption notice periods (in days), the lock-up provisions (in months), and the assets under management (in hundred of millions of dollars). At the end of each year between 1995 and 2005, the value of a portfolio characteristic is defined as the equally-weighted average of the individual hedge funds' characteristics. The numbers in Table 5 correspond to the time series averages of the portfolio characteristics' values over the period 1995 to 2005.

It is clear from Table 5 that, on average, over time the number of hedge funds in the portfolios which exclude managerial skills are much larger for all hedge fund styles. This feature may partially explain the phenomenon documented in the last part of Section 2.5.1, namely that these portfolios are less volatile as they are more diversified. For example, in Emerging Markets, the time series average number of hedge funds for the portfolios ND, PD-1 and PD-2 is always higher than thirty while the average number of hedge funds for the other four portfolios is lower than nine. This observation is not surprising because, as implied by the hedge fund return generating model (2.1), the skeptic and agnostic investors prefer to bet on the specific risks of hedge funds while the dogmatic investors try to diversify these risks.

Hedge funds in the dogmatic portfolios usually tend to be older except that for a few hedge fund styles, hedge funds in the portfolio PS-1 on average have longer records than those selected by some dogmatic portfolios, in other words, the skeptic and agnostic investors prefer younger funds.

An interesting result in Table 5 is that the portfolios taking into account managerial skills do not necessarily choose hedge funds with more restrictive lock-up provisions and redemption notice periods. Aragon (2007) uses hedge fund share restrictions like lock-up provisions and redemption notice periods as transaction cost approximation and documents a positive and concave relation between share restrictions and excess returns on hedge funds. More precisely, he shows that the excess returns of hedge funds with share restrictions are 4-7% higher than those delivered by other hedge funds which do not have such share restrictions. One important implication of these results is that our conclusions about the effect of liquidity risk on hedge fund performance are most likely not driven by hedge funds intrinsic liquidity restrictions.

The relation between the assets under management and the types of portfolios shows no clear pattern. In the case of Event Driven, Emerging Markets, Equity Market Neutral, Long/Short Equity Hedge and Managed Futures style based portfolios, the dogmatic hedge funds generally display more assets under management. However, in the case of Multi-Strategy or Global Macro portfolios, the average assets managed by skeptic and agnostic hedge funds tend to be generally larger.

2.5.3 Performance evaluation results

We evaluate the out-of-sample performance of portfolios first with respect to the Hasanhodzic and Lo (2007) six-factor model, and then using the extended performance evaluation model that is obtained by including the liquidity risk factor $LIQRISK^{ps}$, which is constructed using Pastor and Stambaugh (2003) liquidity measure, into Eq. (2.5).

The performance evaluation results excluding the effect of liquidity risk are reported in the third column of Table 6. First, we corroborate Avramov et al. (2007) empirical results since we also find that the portfolio strategies which incorporate predictability in managerial skills perform in general noticeably better than other strategies. For example, for Emerging Markets hedge funds, the alphas generated by the portfolio strategies PS-1, PA-1 and PA-2 are significant at least at 10% level and about one time higher that the highest alpha generated by the strategies ND, PD-1, PD-2. Moreover, for Long/Short Equity Hedge funds, the significant alphas generated by both the portfolio strategies PS-1 and PA-2 are higher than the highest alpha generated by the strategies ND, PD-1, PD-2 although the difference is quite small. It is worth mentioning that the alphas generated by the portfolio strategies PS-1, PS-2, PA-1 and PA-2 are surprisingly high for Multi-strategy funds, reaching between 1.54% for the PS-2 investor and 2.69% for the PS-1 investor. This may be due to the fact that these hedge funds have the flexibility to opportunistically maneuver between strategies based on the business cycles and can exploit this at an even greater advantage when full business cycle induced predictability is accounted for.

Second, predictability in managerial skills matters the most for portfolio strategies belonging to the Event Driven, Emerging Markets, Equity Market Neutral and Multi-Strategy styles that, except for Equity Market Neutral, can be mapped into the following alternatively defined hedge fund styles: Multi-process and Directional Trader[14]. Avramov et al.

[14]See Appendix A in Agarwal et al. (2005).

(2007) obtained similar results[15]. Note that, in our case, incorporating predictability in managerial skills can even improve the performance of portfolio strategies belonging to the Relative Value (including Convertible Arbitrage, Equity Market Neutral and Fixed Income Arbitrage) and Fund of Funds styles, unlike in Avramov et al. (2007).

Third, compared with the alphas obtained in Avramov et al. (2007), ours are usually higher, particularly for the portfolio strategies PS-1, PS-2, PA-1 and PA-2 belonging to the Relative Value, Fund of Funds and Multi-Strategy styles, this result may be attributed to the steady bull market during the 2003-2006 period which has not been considered in Avramov et al. (2007).

The results obtained so far show that the strategies PS-1, PS-2, PA-1 and PA-2 perform better than the strategies ND, PD-1 and PD-2 in most hedge fund styles, in other words, the strategies that incorporate predictability in managerial skills into their investment decisions are able to generate higher significant excess returns. Thus, it seems that: first, there exist hedge fund managers who possess stock selection and benchmark timing skills; second, investors are able to select the managers with such skills by exploiting business cycle variables.

However, when evaluating the performance of these hedge funds' portfolios, we have until now ignored an important risk to which many hedge funds styles may be exposed, namely liquidity risk. How does liquidity risk affect the performance of these portfolio strategies? Can an omitted illiquidity risk premium explain part of the outperformance of portfolio strategies that incorporate predictability in managerial skills? To answer these questions, let us focus at the fourth column of Table 6, which reports the alphas and the liquidity risk betas of the various hedge fund portfolios under the enlarged performance model with liquidity risk factor $LIQRISK^{ps}$.

The main results can be summarized as follows: the estimated liquidity risk beta β_{ps} is significantly positive at least at the 5% level for almost all portfolio strategies within the Convertible Arbitrage, Event Driven, Emerging Markets, Fund of Funds, Global Macro and Long/Short Equity Hedge strategies, with a unique exception for the portfolio PA-1 in the Emerging Markets. More importantly, once the effect of liquidity risk is introduced, alphas are reduced to insignificant levels for most hedge fund style portfolios, this result holds no matter whether or not predictability in managerial skills is incorporated. Indeed, the number of alphas which are significant at least at 10 % level is reduced from 43 to 22 cases once the effect of liquidity risk is considered.

[15]See the results in Table 3 of Avramov et al. (2007).

To be precise, for hedge funds in Convertible Arbitrage, Fund of Funds, Global Macro and Long/Short Equity Hedge, when the effect of liquidity risk is introduced, the alphas generated by the strategies PS-1, PS-2, PA-1 and PA-2 are generally insignificant. For instance, among Fund of Funds portfolio strategies, the strategy PA-1 delivers an alpha of 1.16%/month which is significant at 10% level and 0.27%/month higher than the highest alpha (0.89%/month and significant at 1% level) generated by the strategies adopted by the dogmatic investors, but this result does not hold any more as soon as the effect of liquidity risk is taken into account. Once again, for Fund of Funds, the alpha generated by the strategy PA-1 is reduced to 0.74%/month and becomes insignificant at conventional levels when liquidity risk premium is accounted for. Liquidity risk bears however a moderate impact on the performance of the strategy PD-2 whose alpha decreases to 0.67%/month, but still remains significant at 1% level. Hence, the introduction of liquidity risk reverses the relative outperformance of these predictability based strategies. Similar results also hold for Convertible Arbitrage, Global Macro and Long/Short Equity Hedge styles based portfolio strategies.

For Event Driven and Emerging Markets hedge funds, although some portfolio strategies incorporating predictability in managerial skills still perform better than other strategies once the effect of liquidity risk is taken into account, the alphas with liquidity risk are dramatically reduced. For example, for Event Driven hedge funds, the alphas generated by the strategies ND, PD-1 and PS-1 turn to be insignificant after the effect of liquidity risk is controlled for. The alpha generated by the strategy PA-2 also decreases by more than 30% (from 1.09%/month to 0.73%/month) although PA-2 is the unique portfolio strategy whose alpha remains significant after considering the effect of liquidity risk.

A possible explanation for these new results is that hedge funds in the Convertible Arbitrage, Event Driven, Emerging Markets, Fund of Funds, Global Macro and Long/Short Equity Hedge styles rely on securities and operate within markets which bear a significant exposure to liquidity risk. Hence, it is not surprising that investors will require a liquidity risk premium if they invest in these types of hedge funds[16].

Finally, it should be noted that, for most portfolio strategies belonging to the Fixed Income Arbitrage, Equity Market Neutral, Managed Futures and Multi-Strategy styles, the effect of liquidity risk is marginal: the estimated liquidity risk beta β_{ps} are, except for the

[16]In a recent paper, Khandani and Lo (2007) analyze long/short equity strategies and find that their liquidity significantly decreased over the last decade. They argue that this may be due to the rapid growth in the number and in the assets under management of long/short equity funds as well as the likely increase in the amount of leverage each fund now employs.

Fixed Income Arbitrage style based portfolios, insignificantly different from zero although positive, and the introduction of liquidity risk only has a rather limited impact on the alphas. For example, in the enlarged performance evaluation model, the alphas generated by all portfolio strategies in Multi-Strategy only decrease by less than 11 basis points which, compared with the levels of the alphas, is very small, and again the skeptic and agnostic portfolio strategies perform much better than the the dogmatic ones. These results suggest that some of these hedge funds do not operate with a large exposure to illiquid markets, this is most likely to be the case for managed futures portfolios, or that, as seems to be the case for Fixed Income Arbitrage and Multi-Strategy portfolios, they are not primarily equity-oriented, and thus do not respond and covary with a liquidity risk factor constructed using stock market data. A noticeable conclusion form our study, is that it is only the Multi-Strategy and the Equity Market Neutral hedge funds portfolios which seem to display consistently significant alphas which furthermore increase when the predictability of managerial skills is accounted for. This conclusion suggests that only managers in these two categories are able to respectively select stocks and time markets predictably and successfully.

2.5.4 Robustness tests

2.5.4.1 Fung and Hsieh (2004) Seven-Factor model

As a robustness test, we first report the results obtained with the Fung and Hsieh (2004) seven-factor model and its extension to account for the effect of liquidity risk when we evaluate the performance of the optimal hedge funds portfolios:

$$\begin{aligned} r_{i,t} = \alpha_i + \beta_{i,1} SP500_t + \beta_{i,2} SCMLC_t + \beta_{i,3} 10Y_t + \beta_{i,4} CredSpr_t \\ + \beta_{i,5} BdOpt_t + \beta_{i,6} FXOpt_t + \beta_{i,7} ComOpt_t + \upsilon_{i,t} \end{aligned} \tag{2.10}$$

where *SP500*=the S&P500 excess return; *SCMLC*=the Wilshire Small Cap 1750 - Wilshire Large Cap 750 return; *10Y*=the month-end to month-end change in the U.S. Federal Reserve 10-year constant-maturity yield; *CredSpr*=the month-end to month-end change in the difference between the Moody's Baa yield and the Federal Reserve's 10-year constant-maturity yield; *BdOp*t=the return of a portfolio of lookback straddles on bond futures; *FXOpt*=the return of a portfolio of lookback straddles on currency futures; *ComOpt*=the return of a portfolio of lookback straddles on commodity futures. The three risk factors of *BdOpt*, *FXOpt* and *ComOpt* are added to capture the fact that hedge fund returns relate to option-based strategy returns[17].

The size factor *SCMLC* and the liquidity risk factor $LIQRISK^{ps}$ are correlated with the coefficient being 0.42, in order to separate the size component from the liquidity risk factor, we regress the liquidity risk factor on the size factor and then use the innovations as the liquidity risk factor proxy denoted by $LIQRISK^{inn}$.

The estimated alphas and liquidity risk betas in this model and its extended version are shown in the fourth column of Table 6. In the Fung and Hsieh (2004) model without $LIQRISK^{inn}$, the alphas α_{fh} of the optimal hedge funds portfolios, compared with those obtained in the Hasanhodzic and Lo (2007) model, are generally lower, particularly, for hedge funds in Global Macro and Managed Futures, no portfolio can generate significant alpha even before controlling for the effect of liquidity risk. For hedge funds in the Fund of Funds and Long/Short Equity Hedge styles, the agnostic and skeptic investors do not significantly outperform the dogmatic investors since none of the portfolios PS-1, PS-2, PA-1 and PA-2 can deliver significant positive alphas.

The second and third sub-columns of the fourth column in Table 6 show the alphas α_{fh}^{inn} and the betas β_{inn}^{fh} of $LIQRISK^{inn}$ in the enlarged Fung and Hsieh (2004) model. Similar as in the previous case, the significance of β_{inn}^{fh} varies across hedge fund styles. For a major number of portfolios belonging to the Convertible Arbitrage, Event Driven, Fund of Funds, Global Macro and Long/Short Equity Hedge styles, β_{inn}^{fh} is significantly positive. In other hedge fund styles, β_{inn}^{fh} is insignificant for most of portfolios. It is easy to observe that the betas of $LIQRISK^{inn}$ are generally lower and less significant in the enlarged Fung and Hsieh (2004) model, this is because the size component is separated from the liquidity risk factor, if we use the liquidity risk factor $LIQRISK^{ps}$ itself rather than its innovation and abstract from the size factor, then we obtain the similar liquidity risk factor betas as in the enlarged Hasanhodzic and Lo (2007) model[18].

In this model, the alphas decrease for a number of hedge fund styles based portfolios when the effect of liquidity risk is taken into account, for example, the alphas generated by the portfolio PA-1 for Convertible Arbitrage hedge funds and the portfolio PA-2 for Emerging Markets hedge funds are reduced to insignificant levels, and the alphas of some portfolios belonging to the Event Driven, Fund of Funds and Long/Short Equity Hedge styles decline by about 20%-30% although they remain significant. Liquidity risk only has limited impact on the alphas of the Equity Market Neutral, Fixed Income Arbitrage and Multi-Strategy styles based portfolios.

[17]To avoid to be voluminous, we do not report the coefficients of *BdOpt*, *FXOpt* and *ComOpt*.

[18]The relevant results are not presented for the sake of parsimony but are available upon request

2.5.4.2 Amihud (2002) liquidity measure

There exist several liquidity risk proxy measures used in the finance literature. It therefore seems reasonable to examine whether our results are robust to alternative liquidity risk measures. In the following, we conduct the previous evaluation exercise using the liquidity risk factor $LIQRISK^{amh}$ constructed with Amihud (2002) liquidity measure for common stocks.

The performance evaluation results for the seven optimal portfolios associated with each hedge funds style are shown in Table 7. The results with the liquidity risk factor $LIQRISK^{amh}$ are generally similar to those that we have obtained above although the effect of introducing the Amihud liquidity risk factor is slightly smaller in the sense that the liquidity risk factor betas are lower and the reduction of the alphas is less pronounced. This is not very surprising since the two liquidity risk factors are highly positive correlated, as documented in Table 3. As before, except for non equity-oriented and market neutral fund portfolios, the coefficients of $LIQRISK^{amh}$ are significantly positive at conventional levels, and the outperformance of the portfolios PS-1, PS-2, PA-1 and PA-2 belonging to the Fund of Funds, Global Macro and Long/Short Equity Hedge styles, after adjusting for liquidity risk premium, vanishes. Again, for the Event Driven and Emerging Markets style portfolios, the alphas generated by the strategies PS-1, PS-2, PA-1 and PA-2 are greatly reduced after liquidity risk is accounted for, but some of them still remain significant. One noticeable different result obtained with $LIQRISK^{amh}$ is that the alpha delivered by the Convertible Arbitrage style based portfolio PA-1 is still significant although it decreases by almost a quarter when the effect of liquidity risk is accounted for.

2.5.4.3 The January effect

According to studies by Keim (1983), Tinic and West (1986), and Eleswarapu and Reinganum (1993), excluding the returns observed during the month of January has the potential of eliminating the impact of liquidity related explanatory factors such as the size and bid-ask spread. In order to check whether our results are driven by a seasonal component, we exclude the month of January from our data and re-evaluate the out-of-sample performance of the hedge funds style based optimal portfolios. The results are displayed in Table 8. The effect of liquidity risk basically remains when we exclude the January data, hence it can be concluded that the January effect is not driving the results observed in our sample.

It is worth mentioning that the alphas generated by most portfolio strategies in most hedge fund styles are reduced when the January data is excluded. For instance, for the

Fund of Funds, Long/Short Equity Hedge and Managed Futures funds, no portfolio strategy incorporating predictability in managerial skills generates a significant alpha even before accounting for the effect of liquidity risk. The lower alphas estimated in the absence of January data may reflect the fact that hedge fund returns are higher in January on average than during the rest of year. Agarwal et al. (2005) show that during 1994-2002, the average January return of hedge funds was indeed higher than the average return during the February-December period (1.65% versus 1.04%) although it was lower than the average December return (1.65% versus 2.52%)[19]. The mere fact that the January and December returns of hedge funds are higher seems to be primarily driven by the tendency of hedge funds to smooth their returns[20].

2.5.5 Hedge fund return smoothing

As already mentioned above, some hedge fund managers have a tendency to smooth returns and this characteristic has been well documented in the literature. Indeed, Asness, Krail, and Liew (2001), Getmansky, Lo, and Makarov (2004) as well as Agarwal and al. (2005) argue that reported hedge fund returns might not reflect all publicly available information that would be reflected in a contemporaneous market index. Instead, hedge fund returns might exhibit-deliberately or involuntarily-delayed informational adjustments and thus correlate with lagged market index returns.

We follow Asness, Krail, and Liew (2001) in that we use a simple method to account for the possible presence of return smoothing and modify the performance evaluation model presented in Section 2.3.5 by adding the lagged S&P500 index returns as an additional factor. The results for this specification of the performance evaluation model are displayed in Table 9[21]. Surprisingly, we observe that the regression coefficients associated with the lagged S&P500 index returns are not necessarily significant nor positive. One potential explanation for this phenomenon is that the effect of the lagged S&P500 index returns is diversified away

[19]See Fig. 1 in Agarwal et al. (2005)

[20]A specific type of return smoothing, which is called "December Bonanza", in the hedge fund industry has been documented in Agarwal et al. (2005) who argue that the unusual higher returns in December may be due to the fact that fund managers manipulate their returns at the end of the year in order to earn incentive fees. In order to account for the effect of this type of return smoothing on the hedge funds portfolio performance, we re-evaluate the out-of-sample performance of the hedge fund style based optimal portfolios while dropping December data, and we find that the results obtained here are quite similar to what we have obtained in Section 2.5.3.3, this means that the alphas are reduced after accounting for this type of hedge fund return smoothing.

[21]We only report the results with one-month lagged S&P500 index returns, the results with longer horizon lagged S&P500 index returns are similar.

at the portfolio level. Therefore, for most types of styles portfolios, the alphas are just slightly reduced and their significances almost remain unchanged after the lagged effect is taken into account. The coefficients of the liquidity risk factor $LIQRISK^{ps}$ are again significantly positive for the portfolios belonging to the Convertible Arbitrage, Event Driven, Emerging Markets, Fund of Funds, Global Macro and Long/Short Equity Hedge styles when the lagged S&P500 index returns are added as an additional factor, and for most portfolio strategies in all hedge fund styles but Equity Market Neutral, Fixed Income Arbitrage, Managed Futures and Multi-Strategy, the alphas are once again reduced to insignificant levels after accounting for the effect of liquidity risk.

2.5.6 Financial crises and the effect of liquidity risk

Financial crises may have an impact on the previous hedge funds portfolio performance results. We pursue this conjecture by examining whether the lack of superior performance documented so far for most hedge funds' portfolios once we introduce liquidity risk is mainly driven by large discontinuous liquidity shocks experienced by hedge funds during financial crises. In order to test this hypothesis, we thus undertake the performance estimation regressions during the sample period while excluding the months of July 1997, August and September 1998, and March 2000 when the following financial crises respectively occurred: the Asian financial crisis, the Russian Government bond default and the following debacle of Long Term Capital Management, and the dot-com bubble burst. The results are displayed in Table 10. This table shows that for a majority of hedge fund styles, the estimated alphas before accounting for the effect of liquidity risk when the return data over the financial crisis periods are excluded are higher than the alphas in Table 6. Indeed, this result is not surprising as most types of style portfolios suffered losses during the financial crises (mainly during the Russian financial crisis and the debacle of Long-Term Capital Management), thus, their estimated alphas are less eroded when the return data over these crises periods is excluded. Moreover, the estimated coefficients of the liquidity risk factor $LIQRISK^{ps}$ are again significantly positive for the hedge fund portfolios belonging to the Convertible Arbitrage, Event Driven, Emerging Markets, Fund of Funds, Global Macro, and Long/Short Equity Hedge styles although they are in general lower than those in Table 6. Third, the effect of liquidity risk on the alphas of all types of style portfolios is almost unchanged. One interesting exception is that now when the effect of liquidity risk is accounted for, the alpha generated by the Managed Futures fund portfolio PA-2 is reduced to an insignificant level while the PD-2's alpha remains significant, which suggests that incorporating predictability in managerial skills could not improve the performance of the Managed Futures style based portfolios when we drop the return data during the financial crisis periods.

Hence, following the results in Table 10, we can conclude that financial crises were not the main liquidity events that affected the performance of hedge fund portfolios in our sample. Frequent yet small liquidity shocks seem to matter as well and to represent an important systematic risk factor affecting most hedge funds portfolios' returns during the sample period.

2.6 Conclusion

In this article, we study the effect of liquidity risk on the performance of optimal hedge fund portfolio strategies. The portfolio strategies in each hedge fund style are formed by incorporating predictability in: (i) managerial skills, (ii) fund risk loadings, and (iii) benchmark returns. Like in Avramov et al. (2007), we observe that, before taking into account the effect of liquidity risk, positive weights-constrained hedge fund portfolios that incorporate predictability in managerial skills dominate other hedge fund styles based portfolios. However, the outperformance of these strategies disappears or weakens dramatically for six out of ten types of hedge funds portfolios as soon as the effect of liquidity risk is incorporated into the performance evaluation framework of Hazanhodzic and Lo (2007). That is, for these hedge fund style based portfolio strategies incorporating predictability in managerial skills, the "alphas" to a large extend merely reflect compensation for liquidity risk bearing. These results are robust to the choice of: (i) an alternative performance evaluation model (The Fung and Hsieh (2004) seven-factor model), (ii) an alternative liquidity risk proxy (based on Amihud illiquidity measure), (iii) the exclusion of the January effect, and (iv) the exclusion of the major recent financial crises.

We believe that these results provide strong support for the fact that many hedge funds act as liquidity providers and should be as such compensated for their exposure to liquidity risk both during and outside of financial crises. This risk bearing compensation is however distinct from hedge funds' managerial ability to generate superior performance and should be recognized as such by incorporating liquidity risk into the performance evaluation models used to identify their skills. Hopefully, using the Pastor and Stambaugh (2003) or the Amihud (2002) liquidity risk proxies advocated in this study may help researchers and practitioners to achieve this goal. However these two liquidity risk proxies were constructed with equity only data and thus may not be suitable for all hedge funds styles. Indeed, we observed that the effect of liquidity risk was very weak for some non-equity (like fixed income arbitrage or multi-strategy) hedge funds and for market neutral hedge funds. This may be attributable in the former case to the fact that these hedge funds do not respond and covary with a liquidity risk factor constructed with equity data. In the future, it would be worth exploring whether similar liquidity risk factors that are constructed with non-equity securities price and volume data can help us explain and disentangle the performance of these hedge funds.

Finally, to the extend that liquidity risk and its premiums may to some extend be predictable, it would be interesting to explore how predictability in managerial skills is affected by the recognition of this additional source of predictability in securities and thus hedge funds' returns. The negative performance during the current dramatic liquidity crisis documented widely in the international press raises doubt regarding the ability of most hedge funds to time and exploit liquidity shocks but such a conjecture certainly deserves further empirical investigation.

2.7 Appendix

The following is a list of hedge fund style descriptions, taken directly from Lipper TASS documentation, that define the criteria used by Lipper TASS in assigning funds in the database to one of eleven possible styles[22].

• Convertible Arbitrage: This strategy is identified by hedge investing in the convertible securities of a company. A typical investment is to be long the convertible bond and short the common stock of the same company. Positions are designed to generate profits from the fixed income security as well as the short sale of stock, while protecting principal from market moves.

• Dedicated Short Bias: Dedicated short sellers were once a robust category of hedge funds before the long bull market rendered the strategy difficult to implement. A new category, short biased, has emerged. The strategy is to maintain net short as opposed to pure short exposure. Short bias managers take short positions in mostly equities and derivatives. The short bias of a manager's portfolio must be constantly greater than zero to be classified in this category.

• Emerging Markets: This strategy involves equity or fixed income investing in emerging markets around the world. Because many emerging markets do not allow short selling, nor offer viable futures or other derivative products with which to hedge, emerging market investing often employs a long-only strategy.

• Equity Market Neutral: This investment strategy is designed to exploit equity market inefficiencies and usually involves being simultaneously long and short matched equity portfolios of the same size within a country. Market neutral portfolios are designed to be either beta or currency neutral, or both. Well designed portfolios typically control for industry, sector, market capitalization, and other exposures. Leverage is often applied to enhance returns.

• Event-Driven: This strategy is defined as equity-oriented investing designed to capture price movement generated by an anticipated corporate event. There are four popular subcategories in event-driven strategies: risk arbitrage, distressed securities, Regulation D and high yield investing.

• Fixed Income Arbitrage: The fixed income arbitrageur aims to profit from price anomalies between related interest rate securities. Most managers trade globally with a goal of generating steady returns with low volatility. This category includes interest rate swap arbitrage, US and non-US government bond arbitrage, forward yield curve arbitrage, and mortgage-backed

[22] Visit http://www.hedgeworld.com for more information.

securities arbitrage. The mortgage-backed market is primarily US-based, over-the-counter and particularly complex.

• Global Macro: Global macro managers carry long and short positions in any of the world's major capital or derivative markets. These positions reflect their views on overall market direction as influence by major economic trends and/or events. The portfolios of these funds can include stocks, bonds, currencies, and commodities in the form of cash or derivatives instruments. Most funds invest globally in both developed and emerging markets.

• Long/Short Equity Hedge: This directional strategy involves equity-oriented investing on both the long and short sides of the market. The objective is not to be market neutral. Managers have the ability to shift from value to growth, from small to medium to large capitalization stocks, and from a net long position to a net short position. Managers may use futures and options to hedge. The focus may be regional, such as long/short US or European equity, or sector specific, such as long and short technology or healthcare stocks. Long/short equity funds tend to build and old portfolios that are substantially more concentrated than those of traditional stock funds.

• Managed Futures: This strategy invests in listed financial and commodity futures markets and currency markets around the world. The managers are usually referred to as Commodity Trading Advisors, or CTAs. Trading disciplines are generally systematic or discretionary. Systematic traders tend to use price and market specific information (often technical) to make trading decisions, while discretionary managers use a judgmental approach.

• Multi-Strategy: The funds in this category are characterized by their ability to dynamically allocate capital among strategies falling within several traditional hedge-fund disciplines. The use of many strategies, and the ability to reallocate capital between them in response to market opportunities, means that such funds are not easily assigned to any traditional category.

• Fund of Funds: Just as the name implies, this is a hedge fund that invests in other hedge funds. Diversification can be across styles by including funds with different strategies, or can be within a single strategy but spread among various hedge funds employing that strategy.

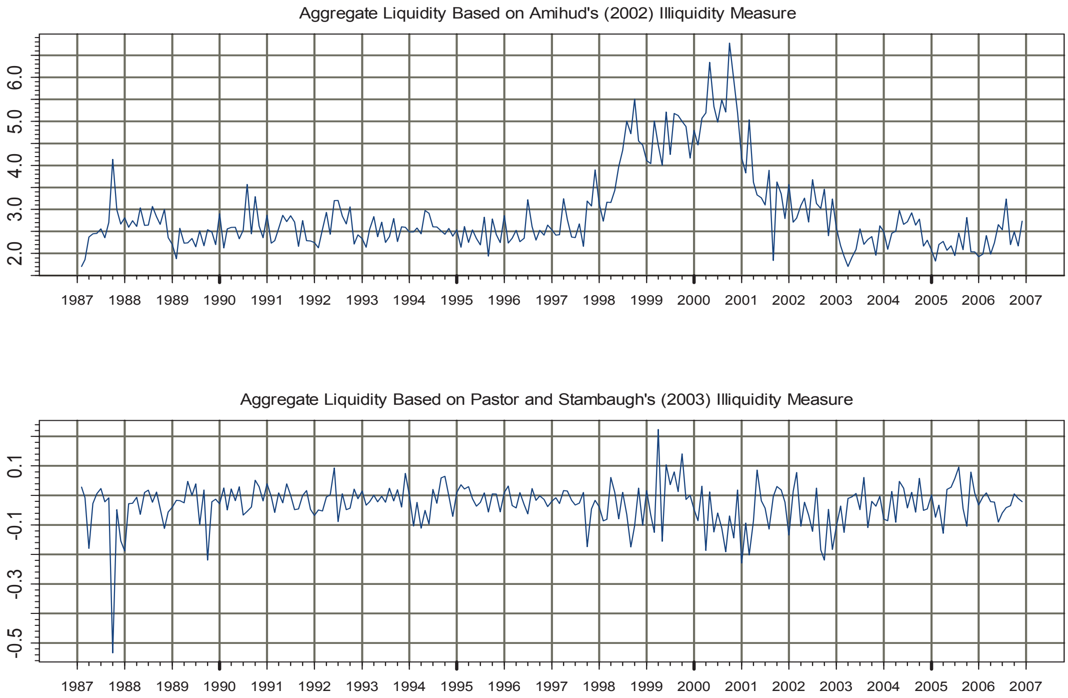

Fig. 1. This figure plots two series of U.S. equity market liquidity levels over the period from January 1987 to December 2006. Each month's observation is obtained by averaging individual stock liquidity measures for the month and then multiplying by m_t/m_1, where m_t is the total dollar value at the end of month t-1 of the stocks included in the average in month t, and month 1 corresponds to August 1962. An individual stock liquidity measure for a given month is either the average of the ratios of the daily absolute return to the trading volume or a regression slope coefficient estimated using daily returns and trading volume data within that month.

Table 1: This table reports some summary statistics for both TASS live and defunct hedge funds over the period between January 1994 and December 2006: sample size, the mean and standard deviation of annualized means, annualized standard deviations, annualized Sharpe ratios and lag-one autocorrelations.

Fund Style	Sample Size	Annualized mean(%)		Annualized stdv(%)		Annualized Sharpe Ratio		ρ_1(%)	
		mean	stdv	mean	stdv	mean	stdv	mean	stdv
Convertible Arbitrage	169	6.50	8.21	6.50	5.15	0.85	2.06	34.55	20.35
Dedicated Short Bias	31	-2.95	8.92	21.57	12.17	-0.23	0.49	7.98	10.59
Event Driven	453	11.22	10.42	8.23	9.27	1.47	2.71	18.94	18.93
Emerging Markets	279	9.88	18.74	20.70	15.32	0.61	1.04	14.32	16.36
Equity Market Neutral	255	6.83	8.23	7.38	5.14	0.65	1.10	2.84	22.16
Fixed Income Arbitrage	204	6.66	7.81	6.41	7.45	1.80	4.62	18.99	21.52
Fund of Funds	1078	7.64	7.63	7.58	7.19	0.95	1.20	17.74	16.16
Global Macro	223	5.61	12.81	13.67	9.59	0.20	0.86	4.85	17.63
Long/Short Equity Hedge	1444	10.16	13.70	15.74	11.79	0.63	0.83	8.41	17.05
Managed Futures	395	5.79	13.83	20.16	19.12	0.20	0.79	-0.10	14.74
Multi-Strategy	167	11.97	13.77	9.51	10.43	1.46	1.90	11.97	23.86
Total	4698	8.60	12.11	12.34	12.08	0.81	1.69	12.25	19.26

Table 2: This table reports some summary statistics *only* for TASS live hedge funds over the period between January 1994 and December 2006: sample size, the mean and standard deviation of annualized means, annualized standard deviations, annualized Sharpe ratios and lag-one autocorrelations.

Fund Style	Sample Size	Annualized mean(%)		Annualized stdv(%)		Annualized Sharpe Ratio		ρ_1(%)	
		mean	stdv	mean	stdv	mean	stdv	mean	stdv
Convertible Arbitrage	76	8.45	5.32	6.20	5.81	1.51	2.53	39.21	21.08
Dedicated Short Bias	16	-5.28	6.52	19.12	10.73	-0.84	1.46	0.00	24.29
Event Driven	271	13.98	12.17	7.10	7.59	2.41	4.58	17.77	21.58
Emerging Markets	164	21.66	21.37	15.21	11.95	1.64	2.45	11.38	20.17
Equity Market Neutral	142	8.91	6.40	6.38	4.88	1.12	1.50	0.19	26.02
Fixed Income Arbitrage	126	9.84	7.25	4.20	3.52	2.55	5.26	15.49	26.20
Fund of Funds	756	10.03	6.98	6.18	4.48	1.88	9.19	17.57	17.61
Global Macro	111	8.76	10.39	10.86	5.98	0.22	1.51	2.12	18.98
Long/Short Equity Hedge	798	14.98	15.65	12.79	7.85	0.99	1.06	6.81	20.68
Managed Futures	162	15.09	32.82	16.95	9.66	1.20	4.63	-1.32	18.09
Multi-Strategy	121	13.17	9.57	7.50	8.67	2.47	4.12	12.67	25.44
Total	2743	12.74	14.71	9.61	8.13	1.54	5.48	11.62	22.05

Table 3
Correlation Matrix of Risk Factors

This table presents the sample correlation matrix of Hasanhodzic and Lo (2007) benchmarks: the excess return on the S&P500 index (SP500), the excess return on the US Dollar index (USDX), the excess return on the Goldman Sachs Commodity Index (GSCI), the excess return on the Lehman Corporate Intermediate Bond Index (LHUSCAAI), the return spread between the US Aggregate Long Credit BAA Bond Index and the Lehman Treasury Long Index (SPREAD), the first difference of the CBOE Volatility Index (FDVIX), and the liquidity risk factors $LIQRISK^{ps}$ and $LIQRISK^{amh}$ that are respectively constructed using Pastor and Stambaugh (2003) and Amihud (2002) liquidity measures.

	SP500	USDX	GSCI	LHUSCAAI	SPREAD	FDVIX	$LIQRISK^{ps}$	$LIQRISK^{amh}$
SP500	1.000							
USDX	0.054	1.000						
GSCI	-0.009	-0.165	1.000					
LHUSCAAI	-0.060	-0.243	0.062	1.000				
SPREAD	-0.474	-0.078	-0.072	-0.262	1.000			
FDVIX	-0.703	-0.092	-0.014	0.098	-0.477	1.000		
$LIQRISK^{ps}$	-0.609	0.045	0.055	0.003	-0.208	0.420	1.000	
$LIQRISK^{amh}$	-0.527	-0.071	0.063	-0.005	-0.122	0.350	0.892	1.000

Table 4

Summary Statistics for Portfolio Performance by Hedge Fund Styles

This table reports, for each hedge fund style, some summary statistics during the period from 1996 to 2006 for the portfolios that are optimal from the perspective of the seven types of investors described above in the context: mean, minimum, maximum, standard deviation, annualized Sharpe ratio, skewness and kurtosis. The portfolios for each type of fund investor are formed by assuming that investor uses the benchmark factor defined as the return on the value-weighted $S\&P500$ index to form expectations about future moments for asset allocation. Investors rebalance their portfolios every 12 months and use the preceding 24 months to form expectations about future moments.

Fund Style	Portfolio Strategy	Mean (%)	Min (%)	Max (%)	Standard Deviation	Sharpe Ratio	Skew	Kurt
Convertible Arbitrage	ND	0.825	−31.28	14.67	5.28	0.340	−1.505	12.99
	PD-1	0.909	−18.07	12.78	4.20	0.495	−1.161	7.81
	PD-2	0.929	−13.53	10.14	3.16	0.682	−1.412	10.01
	PS-1	0.880	−20.33	13.58	4.49	0.440	−1.111	7.88
	PS-2	0.954	−25.71	25.37	5.45	0.410	−0.359	10.26
	PA-1	1.211	−21.99	14.17	4.22	0.736	−1.224	10.86
	PA-2	0.556	−12.96	10.90	3.64	0.236	−0.859	6.11
Event Driven	ND	1.087	−18.85	14.94	3.79	0.711	−0.873	9.07
	PD-1	0.834	−16.25	8.99	2.87	0.636	−1.730	12.41
	PD-2	0.763	−16.21	8.11	2.76	0.570	−2.062	14.06
	PS-1	1.374	−14.15	12.84	5.07	0.726	−0.428	3.54
	PS-2	1.166	−20.00	16.82	5.35	0.554	−0.701	5.89
	PA-1	1.385	−17.11	18.79	6.42	0.578	−0.102	3.88
	PA-2	1.604	−9.41	12.41	3.89	1.149	−0.026	3.32
Emerging Markets	ND	0.832	−47.21	22.54	7.83	0.232	−1.814	13.55
	PD-1	1.093	−34.75	16.57	5.31	0.511	−2.405	18.05
	PD-2	0.959	−32.44	13.44	4.86	0.463	−2.623	19.20
	PS-1	2.104	−42.87	34.94	9.27	0.671	−0.170	7.77
	PS-2	0.777	−22.34	22.25	7.01	0.231	−0.206	4.09
	PA-1	1.879	−14.11	26.08	7.59	0.716	−0.801	4.12
	PA-2	1.978	−39.90	24.15	7.34	0.787	−1.045	10.72
Equity Market Neutral	ND	0.718	−7.33	13.69	3.62	0.394	0.698	4.74
	PD-1	1.050	−5.43	8.91	2.34	1.109	0.128	4.49
	PD-2	0.943	−5.01	9.07	2.18	1.016	0.457	4.87
	PS-1	1.781	−12.39	15.51	4.31	1.187	−0.096	5.11
	PS-2	1.291	−9.44	10.02	3.21	1.061	−0.244	4.05
	PA-1	1.585	−13.34	17.07	4.00	1.102	−0.429	6.82
	PA-2	1.263	−13.78	13.20	3.10	1.068	−0.491	9.89

Table 4 (Cont.)

Fund Style	Portfolio Strategy	Mean (%)	Min (%)	Max (%)	Standard Deviation	Sharpe Ratio	Skew	Kurt
Fund of Funds	ND	1.137	−7.44	15.93	3.99	0.724	0.339	3.77
	PD-1	0.656	−5.82	13.08	2.93	0.414	0.620	5.21
	PD-2	1.055	−6.78	13.31	2.65	0.982	0.529	7.15
	PS-1	0.657	−30.44	18.31	7.19	0.168	−0.724	6.27
	PS-2	0.881	−10.27	17.81	4.00	0.500	1.227	6.80
	PA-1	1.289	−30.44	28.64	6.67	0.511	−0.395	9.95
	PA-2	0.822	−11.85	10.13	3.47	0.514	−0.188	4.39
Fixed Income Arbitrage	ND	0.667	−9.58	13.61	2.77	0.446	0.213	7.36
	PD-1	0.636	−23.42	26.93	3.76	0.302	0.018	34.17
	PD-2	0.728	−23.17	26.93	3.87	0.376	−0.009	30.27
	PS-1	0.912	−6.48	12.44	2.78	0.754	0.404	5.45
	PS-2	0.738	−12.86	15.83	3.41	0.437	−0.479	8.95
	PA-1	0.996	−8.17	13.67	3.07	0.778	0.335	5.48
	PA-2	0.607	−14.34	15.28	3.37	0.308	−0.652	9.72
Global Macro	ND	0.910	−15.63	16.80	5.15	0.405	−0.217	4.61
	PD-1	0.833	−8.24	11.15	3.82	0.475	0.326	3.46
	PD-2	0.883	−12.79	12.93	3.69	0.542	0.074	5.53
	PS-1	0.816	−12.77	18.81	5.23	0.134	0.262	3.51
	PS-2	0.873	−21.41	18.71	6.05	0.506	0.414	5.01
	PA-1	0.512	−11.87	16.93	5.32	0.338	0.574	5.31
	PA-2	1.151	−16.60	22.47	5.78	0.324	−0.299	5.48
Long/Short Equity Hedge	ND	1.249	−31.34	17.23	6.44	0.507	−0.688	7.61
	PD-1	1.026	−10.10	11.16	3.46	0.724	0.095	4.42
	PD-2	1.448	−10.92	10.91	3.25	1.224	−0.147	5.15
	PS-1	1.474	−14.86	28.69	6.74	0.600	0.831	5.20
	PS-2	1.202	−42.69	102.8	12.19	0.254	3.993	39.42
	PA-1	1.052	−11.96	20.11	5.44	0.473	0.733	4.39
	PA-2	1.338	−14.96	17.31	5.54	0.644	−0.177	3.52
Managed Futures	ND	1.071	−13.71	20.96	6.09	0.435	0.412	3.76
	PD-1	0.785	−14.45	15.72	4.57	0.362	0.126	4.34
	PD-2	1.043	−15.83	21.83	4.78	0.535	0.565	7.03
	PS-1	0.558	−21.29	35.14	6.97	0.125	0.649	7.42
	PS-2	0.355	−23.79	64.09	9.13	0.018	2.607	20.16
	PA-1	0.743	−23.61	53.90	8.33	0.182	1.811	14.61
	PA-2	1.086	−19.35	33.51	7.14	0.379	0.754	6.62
Multi-Strategy	ND	1.222	−17.63	13.25	4.47	0.710	−0.359	5.27
	PD-1	1.407	−17.63	11.42	3.33	1.146	−1.022	10.57
	PD-2	1.361	−16.01	10.91	3.40	1.071	−0.928	8.48
	PS-1	3.222	−17.63	61.87	8.14	1.236	2.951	22.57
	PS-2	2.290	−17.63	51.36	7.07	0.970	2.592	19.79
	PA-1	3.133	−17.63	83.13	9.79	0.997	4.247	35.40
	PA-2	3.220	−11.65	101.8	9.72	1.036	7.978	81.71

Table 5

Analysis of the Optimal Portfolios' Components

This table reports summary statistics of some characteristics variables of all the seven portfolios within each hedge fund investment style, namely the characteristics variables are: the number of hedge funds in a portfolio, the age of hedge funds since their inceptions (in years), redemption notice periods (in days), lock-up provisions (in months), and the assets under management (in hundred millions). At the end of each year between 1995 and 2005, the value of s portfolio characteristics variable is defined as the equally-weighted average of individual characteristics variable values in this portfolio. The numbers in this table correspond the time series averages of portfolio characteristics variable values over the period from 1995 to 2005.

Portfolio Strategy	Characteristics	Convertible Arbitrage	Event Driven	Emerging Markets	Equity Market Neutral	Fund of Funds	Fixed Income Arbitrage	Global Macro	Long/Short Equity Hedge	Managed Futures	Multi Strategy
	Number	16.1	37.5	33.6	16.8	80.5	11.6	14.9	114.9	33.9	14.7
	Age	5.86	5.78	5.41	3.87	5.71	3.99	5.74	5.31	7.05	5.43
ND	Redemption Notice Period	31.5	42.5	28.3	28.3	34.4	22.5	19.9	34.7	9.61	35.4
	Lock-up Period	1.45	5.65	1.41	2.26	1.22	1.36	1.19	4.56	1.49	3.27
	Assets under Management	1.14	1.67	1.11	0.89	0.81	1.09	3.29	1.21	0.34	0.90
	Number	18.5	32.4	44.9	23.7	68.3	13.8	15.5	164.5	42.5	15.8
	Age	5.18	4.99	5.17	4.09	4.96	4.12	5.61	4.81	6.62	4.92
PD-1	Redemption Notice Period	36.7	48.4	29.7	26.7	40.9	26.7	19.6	32.5	10.4	32.9
	Lock-up Period	2.24	5.99	2.02	2.25	0.99	1.20	1.54	7.80	0.99	3.74
	Assets under Management	1.29	1.48	1.15	0.85	0.72	1.26	1.68	0.91	0.37	2.75
	Number	12.6	22.9	37.6	16.1	52.0	13.0	19.4	135.4	60.4	11.2
	Age	4.96	4.87	5.14	4.21	4.73	4.52	5.79	5.24	6.58	4.90
PD-2	Redemption Notice Period	36.9	49.9	30.9	29.9	43.2	25.8	18.8	33.8	10.6	40.8
	Lock-up Period	2.22	7.10	1.86	1.99	1.42	1.31	1.33	4.99	0.67	3.48
	Assets under Management	1.25	1.19	0.98	1.12	1.02	2.43	1.74	0.99	0.39	5.07

Table 5 (Cont.)

Portfolio Strategy	Characteristics	Convertible Arbitrage	Event Driven	Emerging Markets	Equity Market Neutral	Fund of Funds	Fixed Income Arbitrage	Global Macro	Long/Short Equity Hedge	Managed Futures	Multi Strategy
	Number	2.73	3.91	6.09	2.82	3.55	1.91	3.82	5.91	6.36	2.27
	Age	5.52	5.10	4.97	3.58	5.19	3.48	5.78	4.71	6.84	5.44
PS-1	Redemption Notice Period	34.5	40.7	30.5	32.8	37.0	23.5	20.7	35.6	13.4	26.9
	Lock-up Period	2.07	7.50	1.99	1.82	1.27	0.55	1.56	5.28	1.46	2.09
	Assets under Management	0.66	1.21	0.70	0.51	0.33	0.76	4.62	0.99	0.12	5.75
	Number	2.00	3.27	4.09	2.09	3.64	2.27	3.09	3.18	5.18	2.36
	Age	4.98	4.51	4.47	3.31	4.54	3.53	4.72	3.57	5.45	3.84
PS-2	Redemption Notice Period	35.2	42.2	35.4	31.5	37.8	25.7	19.7	34.9	15.4	26.1
	Lock-up Period	3.45	6.54	3.49	3.18	3.11	1.09	1.46	4.05	0.55	1.65
	Assets under Management	0.78	1.18	0.67	0.58	0.46	1.07	1.34	1.04	0.12	2.29
	Number	1.82	2.91	3.64	2.18	2.18	2.00	2.64	5.00	4.45	1.82
	Age	4.09	4.10	4.05	3.17	4.31	3.39	5.08	3.35	5.83	4.67
PA-1	Redemption Notice Period	35.2	37.8	23.3	29.9	32.1	24.7	18.2	32.1	14.6	32.1
	Lock-up Period	3.45	8.18	1.06	2.40	0.55	0.55	1.09	5.11	1.47	3.09
	Assets under Management	0.56	1.16	0.57	0.58	1.11	0.99	4.21	0.75	0.17	5.73
	Number	3.73	5.91	8.64	4.00	6.27	3.36	5.55	14.5	10.7	3.82
	Age	4.63	4.18	4.66	3.40	4.06	3.80	5.11	3.75	6.17	4.41
PA-2	Redemption Notice Period	40.6	38.3	31.9	30.0	34.8	34.3	18.9	33.6	12.1	29.1
	Lock-up Period	4.21	6.46	2.11	2.25	3.68	0.84	1.86	5.67	0.38	2.56
	Assets under Management	1.25	1.17	0.72	0.59	1.00	1.21	4.24	0.72	0.13	2.67

Table 6

Out-of-sample Performance of Portfolio Strategies by Hedge Fund Styles

This table reports, for each hedge fund style, the alphas of portfolios which are optimal from the perspective of the seven types of investors described above in the context and the coefficients of liquidity risk factor $LIQRISK^{ps}$ (or $LIQRISK^{inn}$) that is constructed using Pastor and Stambaugh (2003) liquidity measure. α_{hl} is the intercept obtained by regressing portfolio excess returns on the Hasanhodzic and Lo (2007) Benchmarks, α_{fh} is the intercept obtained by regressing portfolio excess returns on the Fung and Hsieh (2004) Benchmarks, α_{hl}^{ps} and α_{fh}^{inn} are the same intercepts, but adjusted for liquidity risk premium. β_{ps}^{hl} and β_{inn}^{fh} are the coefficients of the liquidity risk factor in the extended Hasanhodzic and Lo (2007) and Fung and Hsieh (2004) models. The symbols $***$, $**$ and $*$ respectively reflect significance at the 1%, 5% and 10% levels.

Fund Style	Portfolio Strategy	Hasanhodzic and Lo Six-Factor Model α_{hl}	α_{hl}^{ps}	β_{ps}^{hl}	Fung and Hsieh Seven-Factor Model α_{fh}	α_{fh}^{inn}	β_{inn}^{fh}
	ND	0.35	-0.18	0.65***	-0.06	-0.45	0.59***
	PD-1	0.53	0.05	0.58***	0.24	-0.08	0.47***
	PD-2	0.58**	0.29	0.35***	0.41	0.18	0.35***
Convertible Arbitrage	PS-1	0.50	0.07	0.52***	0.23	-0.04	0.40***
	PS-2	0.70	0.23	0.56***	0.28	-0.02	0.45***
	PA-1	0.86**	0.55	0.38***	0.63*	0.39	0.37***
	PA-2	0.20	-0.10	0.37***	-0.01	-0.21	0.31***
	ND	0.59**	0.30	0.35***	0.33	0.19	0.20**
	PD-1	0.42*	0.13	0.35***	0.20	0.04	0.24***
	PD-2	0.39	0.07	0.38***	0.17	-0.03	0.30***
Event Driven	PS-1	0.76*	0.24	0.63***	0.63*	0.34	0.44***
	PS-2	0.46	-0.03	0.58***	0.43	0.16	0.41***
	PA-1	0.79	0.32	0.57***	0.69	0.42	0.39*
	PA-2	1.09***	0.73**	0.44***	1.01***	0.79**	0.31***
	ND	0.31	-0.29	0.72***	-0.17	-0.52	0.52**
	PD-1	0.75	0.37	0.46***	0.25	0.05	0.30*
	PD-2	0.66	0.37	0.34***	0.16	0.04	0.17
Emerging Markets	PS-1	1.73**	1.16	0.68***	0.91	0.67	0.36
	PS-2	0.62	0.26	0.44**	-0.11	-0.09	-0.03
	PA-1	1.37*	1.16	0.25	1.00	1.04	-0.06
	PA-2	1.61**	1.12*	0.59***	0.94*	0.70	0.35
	ND	0.33	-0.01	0.40***	0.14	-0.11	0.38***
	PD-1	0.81***	0.77***	0.04	0.65***	0.60***	0.08
	PD-2	0.69***	0.62***	0.08	0.57***	0.49***	0.12*
Equity Market Neutral	PS-1	1.72***	1.57***	0.18	1.42***	1.34***	0.12
	PS-2	1.25***	1.21***	0.05	0.92***	0.84***	0.11
	PA-1	1.49***	1.40***	0.11	1.22***	1.10***	0.18
	PA-2	1.00***	0.94***	0.08	0.98***	0.80***	0.26***

Table 6 (Cont.)

Fund Style	Portfolio Strategy	Hasanhodzic and Lo Six-Factor Model			Fung and Hsieh Seven-Factor Model		
		α_{hl}	α_{hl}^{ps}	β_{ps}^{hl}	α_{fh}	α_{fh}^{ps}	β_{ps}^{fh}
Fund of Funds	ND	0.87**	0.65*	0.26**	0.62**	0.45	0.26**
	PD-1	0.38	0.14	0.29***	0.14	-0.02	0.24***
	PD-2	0.89***	0.67***	0.26***	0.60***	0.39*	0.31***
	PS-1	0.53	-0.17	0.84***	-0.02	-0.41	0.58**
	PS-2	0.59*	0.39	0.24**	0.38	0.29	0.13
	PA-1	1.16*	0.74	0.50***	0.73	0.50	0.34
	PA-2	0.62*	0.38	0.29***	0.44	0.34	0.15
Fixed Income Arbitrage	ND	0.47*	0.32	0.18**	0.18	0.11	0.11
	PD-1	0.20	0.10	0.11	0.11	0.09	0.02
	PD-2	0.32	0.22	0.13	0.24	0.20	0.06
	PS-1	0.73***	0.68**	0.06	0.56**	0.50**	0.09
	PS-2	0.68**	0.51	0.21**	0.28	0.17	0.17
	PA-1	0.82***	0.77**	0.05	0.64**	0.59**	0.08
	PA-2	0.45	0.29	0.20**	0.25	0.13	0.17
Global Macro	ND	0.95**	0.58	0.44***	0.19	-0.02	0.31**
	PD-1	0.65*	0.25	0.48***	0.26	0.03	0.34***
	PD-2	0.79**	0.46	0.41***	0.37	0.10	0.40***
	PS-1	0.41	0.06	0.43***	-0.18	-0.37	0.29*
	PS-2	0.77	0.41	0.44***	0.47	0.32	0.22
	PA-1	0.64	0.28	0.43***	0.27	0.03	0.35*
	PA-2	1.20**	0.78	0.51***	0.34	0.09	0.38*
Long/Short Equity Hedge	ND	0.95*	0.50	0.55***	0.27	0.09	0.27
	PD-1	0.53*	0.23	0.36***	0.33	0.19	0.21**
	PD-2	1.01***	0.69***	0.39***	0.82***	0.59**	0.34***
	PS-1	1.15*	0.46	0.84***	0.62	0.40	0.32*
	PS-2	1.35	0.69	0.79**	0.21	0.05	0.24
	PA-1	0.49	0.09	0.48***	0.62	0.47	0.22
	PA-2	1.02**	0.53	0.59***	0.60	0.38	0.31*
Managed Futures	ND	0.94*	0.74	0.24	0.67	0.48	0.28
	PD-1	0.63	0.55	0.10	0.35	0.28	0.11
	PD-2	1.04**	0.92**	0.14	0.63	0.51	0.18
	PS-1	0.72	0.60	0.16	0.39	0.19	0.28
	PS-2	0.73	0.70	0.03	0.54	0.45	0.13
	PA-1	0.91	0.97	-0.07	0.76	0.76	0.00
	PA-2	1.28*	1.18*	0.12	0.90	0.78	0.18
Multi-Strategy	ND	0.85***	0.71**	0.18*	0.47	0.27	0.28**
	PD-1	0.99***	0.88***	0.14*	0.77***	0.66***	0.17*
	PD-2	0.91***	0.81***	0.13	0.77***	0.71***	0.09
	PS-1	2.69***	2.65***	0.04	2.43***	2.51***	-0.12
	PS-2	1.54**	1.48**	0.08	1.64***	1.72***	-0.12
	PA-1	2.58***	2.52***	0.07	2.33***	2.43***	-0.16
	PA-2	2.65***	2.54***	0.13	2.54***	2.64***	-0.15

Table 7
Amihud (2002) Liquidity Risk Factor

This table reports, for each hedge fund style, the alphas of portfolios which are optimal from the perspective of the seven types of investors described above in the context and the coefficients of liquidity risk factor $LIQRISK^{amh}$ that is constructed using Amihud (2002) liquidity measure. α is the intercept obtained by regressing portfolio excess returns on the Hasanhodzic and Lo (2007) Benchmarks, and α_{amh} is the same intercept, but adjusted for liquidity risk premium. β_{amh} is the coefficient of liquidity risk factor in the extended Hasanhodzic and Lo (2007) model. The symbols $***$, $**$ and $*$ respectively reflect significance at the 1%, 5% and 10% levels.

Fund Style		Portfolio Strategy						
		ND	PD-1	PD-2	PS-1	PS-2	PA-1	PA-2
	α	0.35	0.53	0.58**	0.50	0.70	0.86**	0.20
Arbitrage Convertible	α_{amh}	0.01	0.21	0.40	0.22	0.42	0.67*	-0.01
	β_{amh}	0.48***	0.43***	0.26***	0.38***	0.38***	0.27***	0.29***
	α	0.59**	0.42*	0.39	0.76*	0.46	0.79	1.09***
Event Driven	α_{amh}	0.41	0.21	0.15	0.44	0.13	0.50	0.86**
	β_{amh}	0.26***	0.29***	0.33***	0.44***	0.46***	0.41***	0.31***
	α	0.31	0.75	0.66	1.73**	0.62	1.37*	1.61**
Emerging Markets	α_{amh}	-0.11	0.49	0.44	1.33*	0.35	1.22*	1.26**
	β_{amh}	0.57***	0.35***	0.30**	0.55***	0.38**	0.21	0.48***
	α	0.33	0.81***	0.69***	1.72***	1.25***	1.49***	1.00***
Equity Market Neutral	α_{amh}	0.13	0.79***	0.65***	1.60***	1.18***	1.37***	0.98***
	β_{amh}	0.27***	0.02	0.05	0.16	0.10	0.16	0.02
	α	0.87**	0.38	0.89***	0.53	0.59*	1.16***	0.62**
Fund of Funds	α_{amh}	0.76**	0.24	0.75***	0.08	0.44	0.86	0.45
	β_{amh}	0.14*	0.19***	0.18***	0.62***	0.21**	0.41***	0.24***
	α	0.47*	0.20	0.32	0.73***	0.68**	0.82***	0.45
Fixed Income Arbitrage	α_{amh}	0.36	0.13	0.24	0.68**	0.54	0.77**	0.31
	β_{amh}	0.15**	0.09	0.11	0.08	0.19**	0.07	0.20**
	α	0.95**	0.65*	0.79**	0.41	0.77	0.64	1.20**
Global Macro	α_{amh}	0.71*	0.39	0.59*	0.19	0.54	0.43	0.85
	β_{amh}	0.33***	0.37***	0.28***	0.30**	0.32**	0.29**	0.48***
	α	0.95*	0.53*	1.01***	1.15*	1.35	0.49	1.02**
Long/Short Equity Hedge	α_{amh}	0.68	0.32	0.81***	0.62	0.91	0.21	0.65
	β_{amh}	0.38***	0.29***	0.28***	0.73***	0.62**	0.39***	0.50***
	α	0.94*	0.63	1.04**	0.72	0.73	0.91	1.28*
Managed Futures	α_{amh}	0.86	0.60	0.97**	0.60	0.66	0.91	1.22*
	β_{amh}	0.11	0.04	0.10	0.17	0.10	-0.01	0.08
	α	0.85***	0.99***	0.91***	2.69***	1.54***	2.58***	2.65***
Multi-Strategy	α_{amh}	0.83**	0.94***	0.82***	2.63***	1.43**	2.49***	2.48***
	β_{amh}	0.04	0.07	0.13*	0.08	0.16	0.13	0.24

Table 8
The Exclusion of the January Effect

This table reports, for each hedge fund style, the alphas of portfolios which are optimal from the perspective of the seven types of investors described above in the context and the coefficients of liquidity risk factor $LIQRISK^{ps}$ that is constructed using Pastor and Stambaugh (2003) liquidity measure. α is the intercept obtained by regressing portfolio excess returns on the Hasanhodzic and Lo (2007) Benchmarks, α_{ps} is the same intercept, but adjusted for liquidity risk premium. β_{ps} is the coefficient of the liquidity risk factor in the extended Hasanhodzic and Lo (2007) model. Data in every January are dropped to exclude the January effect as documented in Keim (1983), Tinic and West (1986) and Eleswarapu and Reinganum (1993). The symbols $***$, $**$ and $*$ respectively reflect significance at the 1%, 5% and 10% levels.

Fund Style		Portfolio Strategy						
		ND	PD-1	PD-2	PS-1	PS-2	PA-1	PA-2
Convertible Arbitrage	α	0.14	0.29	0.40	0.35	0.55	0.73*	0.05
	α_{ps}	-0.27	-0.07	0.18	0.02	0.21	0.49	-0.19
	β_{ps}	0.69***	0.59***	0.36***	0.55***	0.57***	0.39***	0.41***
Event Driven	α	0.55*	0.33	0.28	0.71	0.35	0.89	1.10***
	α_{ps}	0.34	0.13	0.05	0.32	0.00	0.52	0.82**
	β_{ps}	0.35***	0.34***	0.37***	0.64***	0.59***	0.61***	0.47***
Emerging Markets	α	0.18	0.61	0.55	1.30	0.56	1.15	1.52**
	α_{ps}	-0.25	0.34	0.35	0.89	0.27	0.99	1.14*
	β_{ps}	0.71***	0.45***	0.32**	0.69**	0.49**	0.28	0.63***
Equity Market Neutral	α	0.31	0.78***	0.71***	1.75***	1.22***	1.54***	1.05***
	α_{ps}	0.07	0.77***	0.67***	1.68***	1.21***	1.49***	1.03***
	β_{ps}	0.39***	0.02	0.06	0.12	0.01	0.07	0.02
Fund of Funds	α	0.68*	0.13	0.79***	0.16	0.39	0.93	0.49
	α_{ps}	0.53	-0.04	0.62**	-0.30	0.23	0.61	0.34
	β_{ps}	0.25**	0.28***	0.28***	0.76***	0.25**	0.52***	0.25**
Fixed Income Arbitrage	α	0.42	0.17	0.29	0.52**	0.52	0.59**	0.26
	α_{ps}	0.32	0.10	0.21	0.51**	0.39	0.58*	0.15
	β_{ps}	0.17**	0.11	0.13	0.03	0.22**	0.02	0.16*
Global Macro	α	0.75*	0.43	0.55	0.35	0.64	0.63	0.98*
	α_{ps}	0.48	0.12	0.30	0.09	0.33	0.35	0.61
	β_{ps}	0.44***	0.51***	0.42***	0.44***	0.52***	0.45***	0.62***
Long/Short Equity Hedge	α	0.82	0.41	0.98***	0.99	1.32	0.26	0.64
	α_{ps}	0.48	0.18	0.73***	0.48	0.78	-0.03	0.28
	β_{ps}	0.58***	0.38***	0.41***	0.86***	0.89**	0.48***	0.60***
Managed Futures	α	1.05*	0.65	1.12**	0.35	0.35	0.58	1.11
	α_{ps}	0.89	0.59	1.03**	0.28	0.35	0.66	1.04
	β_{ps}	0.26*	0.11	0.15	0.11	-0.01	-0.13	0.12
Multi-Strategy	α	0.82**	0.93***	0.85***	2.70***	1.43**	2.51**	2.74***
	α_{ps}	0.72**	0.85***	0.79***	2.68***	1.42**	2.48**	2.67***
	β_{ps}	0.18*	0.14*	0.09	0.03	0.02	0.05	0.12

Table 9
Adjusting for Hedge Fund Return Smoothing

This table reports, for each hedge fund style, the alphas of portfolios which are optimal from the perspective of the seven types of investors described above in the context, the coefficients of lag-one market returns, and the coefficients of the liquidity risk factor $LIQRISK^{ps}$ that is constructed using Pastor and Stambaugh (2003) liquidity measure. α is the intercept obtained by regressing portfolio excess returns on the Hasanhodzic and Lo (2007) Benchmarks, α_{lag} is the same intercept, but adjusted for the additional explanatory variable of lag-one market returns, and α^{ps}_{lag} further adjusted for liquidity risk premium. β_{lag} is the coefficient of lag-one market returns, and β^{ps}_{lag} is the same coefficient, but adjusted for the effect of liquidity risk. β_{ps} is the coefficient of the liquidity risk factor in the extended Hasanhodzic and Lo (2007) model. The symbols $***$, $**$ and $*$ respectively reflect significance at the 1%, 5% and 10% levels.

	Convertible Arbitrage							Event Driven						
	ND	PD-1	PD-2	PS-1	PS-2	PA-1	PA-2	ND	PD-1	PD-2	PS-1	PS-2	PA-1	PA-2
α	0.35	0.53	0.58**	0.50	0.70	0.86**	0.20	0.59**	0.42*	0.39	0.76*	0.46	0.79	1.09***
α_{lag}	0.29	0.51	0.55*	0.51	0.70	0.86**	0.21	0.55*	0.39	0.34	0.73	0.44	0.78	1.02***
β_{lag}	0.19*	0.06	0.11*	-0.04	-0.01	0.00	-0.03	0.14**	0.10*	0.13**	0.11	0.07	0.04	0.22***
α^{ps}_{lag}	-0.19	0.05	0.29	0.07	0.24	0.55	-0.10	0.30	0.13	0.07	0.24	-0.02	0.32	0.72**
β^{ps}_{lag}	0.08	-0.05	0.05	-0.14	-0.12	-0.07	-0.10	0.08	0.04	0.07	-0.01	-0.04	-0.06	0.14*
β_{ps}	0.63***	0.60***	0.34***	0.57***	0.60***	0.41***	0.40***	0.33***	0.33***	0.36***	0.63***	0.60***	0.59***	0.39***
	Emerging Markets							Equity Market Neutral						
	ND	PD-1	PD-2	PS-1	PS-2	PA-1	PA-2	ND	PD-1	PD-2	PS-1	PS-2	PA-1	PA-2
α	0.31	0.75	0.66	1.73**	0.62	1.37*	1.61**	0.33	0.81***	0.69***	1.72***	1.25***	1.49**	1.00***
α_{lag}	0.23	0.70	0.58	1.61**	0.55	1.31*	1.53**	0.29	0.79***	0.67***	1.65***	1.19***	1.40***	0.95***
β_{lag}	0.23	0.15	0.23**	0.37**	0.21	0.19	0.24*	0.10	0.04	0.06	0.20**	0.17**	0.27***	0.16**
α^{ps}_{lag}	-0.29	0.36	0.37	1.15	0.25	1.15	1.11*	-0.01	0.77***	0.62***	1.56***	1.20***	1.39***	0.93***
β^{ps}_{lag}	0.11	0.07	0.18*	0.26	0.14	0.15	0.14	0.02	0.04	0.05	0.18*	0.17**	0.27***	0.16**
β_{ps}	0.68***	0.43***	0.28**	0.60**	0.39**	0.20	0.55***	0.39***	0.03	0.06	0.12	-0.01	0.02	0.02

Table 9 (Cont.)

	Fund of Funds							Fixed Income Arbitrage						
	ND	PD-1	PD-2	PS-1	PS-2	PA-1	PA-2	ND	PD-1	PD-2	PS-1	PS-2	PA-1	PA-2
α	0.87**	0.38	0.89***	0.53	0.59*	1.16*	0.62**	0.47*	0.20	0.32	0.73***	0.68**	0.82***	0.45
α_{lag}	0.88**	0.37	0.88***	0.51	0.56*	1.14*	0.65*	0.44*	0.11	0.25	0.70**	0.66**	0.79***	0.43
β_{lag}	-0.03	0.03	0.03	0.06	0.09	0.05	-0.09	0.09	0.28***	0.24***	0.09	0.06	0.10	0.09
α_{lag}^{ps}	0.65*	0.14	0.67***	-0.16	0.39	0.74	0.38	0.32	0.09	0.21	0.67**	0.50	0.77**	0.29
β_{lag}^{ps}	-0.09	-0.02	-0.02	-0.10	0.05	-0.04	-0.15**	0.07	0.28***	0.23***	0.09	0.03	0.10	0.05
β_{ps}	0.29***	0.29***	0.27***	0.87***	0.22**	0.52***	0.34***	0.16**	0.02	0.05	0.03	0.20**	0.02	0.18*
	Global Macro							Long/Short Equity Hedge						
	ND	PD-1	PD-2	PS-1	PS-2	PA-1	PA-2	ND	PD-1	PD-2	PS-1	PS-2	PA-1	PA-2
α	0.95**	0.65*	0.79**	0.41	0.77	0.64	1.20**	0.95*	0.53*	1.01***	1.15*	1.35	0.49	1.02**
α_{lag}	0.91**	0.60*	0.74**	0.41	0.77	0.63	1.26**	0.94*	0.51*	0.98***	1.14*	1.28	0.50	0.96**
β_{lag}	0.11	0.15**	0.16**	0.01	0.02	0.05	-0.18	0.02	0.04	0.10	0.05	0.23	-0.02	0.17*
α_{lag}^{ps}	0.58	0.25	0.45	0.06	0.41	0.28	0.79	0.50	0.23	0.68***	0.46	0.69	0.10	0.53
β_{lag}^{ps}	0.03	0.07	0.09	-0.07	-0.07	-0.03	-0.29**	-0.08	-0.03	0.03	-0.11	0.09	-0.11	0.07
β_{ps}	0.44***	0.46***	0.38***	0.45***	0.46***	0.45***	0.61***	0.58***	0.37***	0.38***	0.88***	0.76**	0.52***	0.56***
	Managed Futures							Multi-Strategy						
	ND	PD-1	PD-2	PS-1	PS-2	PA-1	PA-2	ND	PD-1	PD-2	PS-1	PS-2	PA-1	PA-2
α	0.94*	0.63	1.04**	0.72	0.73	0.91	1.28*	0.85***	0.99***	0.91***	2.69***	1.54**	2.58***	2.65***
α_{lag}	0.93*	0.62	1.04**	0.73	0.78	0.95	1.28*	0.88***	0.99***	0.88***	2.73***	1.56**	2.63***	2.71***
β_{lag}	0.03	0.03	0.00	-0.03	-0.15	-0.13	-0.01	-0.08	0.03	0.10	-0.14	-0.04	-0.16	-0.18
α_{lag}^{ps}	0.74	0.55	0.93**	0.60	0.71	0.97	1.18*	0.71**	0.88***	0.81***	2.66***	1.48**	2.53***	2.55***
β_{lag}^{ps}	-0.01	0.02	-0.03	-0.06	-0.16	-0.12	-0.04	-0.12	0.01	0.08	-0.16	-0.06	-0.19	-0.21
β_{ps}	0.25	0.09	0.15	0.18	0.08	-0.03	0.14	0.22**	0.14*	0.10	0.10	0.10	0.13	0.20

Table 10
Financial Crises and the Effect of Liquidity Risk

This table reports, for each hedge fund style, the alphas of portfolios which are optimal from the perspective of the seven types of investors described above in the context and the coefficients of the liquidity risk factor $LIQRISK^{ps}$ that is constructed using Pastor and Stambaugh (2003) liquidity measure. α is the intercept obtained regressing portfolio excess returns on the Hasanhodzic and Lo (2007) Benchmarks, α_{ps} is the same intercept, but adjusted for liquidity risk premium. β_{ps} is the coefficient of the liquidity risk factor in the extended Hasanhodzic and Lo (2007) model. Data in the months of 7/1997, 8-9/1998 and 3/2000 are dropped. The symbols $***$, $**$ and $*$ respectively reflect significance at the 1%, 5% and 10% levels.

Fund Style		Portfolio Strategy						
		ND	PD-1	PD-2	PS-1	PS-2	PA-1	PA-2
Convertible Arbitrage	α	0.52	0.63*	0.69***	0.55	0.74	0.92**	0.25
	α_{ps}	0.08	0.20	0.46*	0.16	0.30	0.64*	-0.01
	β_{ps}	0.53***	0.51***	0.27***	0.47***	0.52***	0.33***	0.31***
Event Driven	α	0.74***	0.55**	0.51**	0.84*	0.49	0.95	1.25***
	α_{ps}	0.47*	0.30	0.24	0.29	-0.03	0.45	0.91***
	β_{ps}	0.32***	0.30***	0.32***	0.66***	0.62***	0.59***	0.41***
Emerging Markets	α	0.70	0.88**	0.79**	1.75**	0.72	1.32*	1.82***
	α_{ps}	0.28	0.61	0.61*	1.34*	0.41	1.16	1.44**
	β_{ps}	0.50***	0.33***	0.21*	0.50**	0.37*	0.19	0.46**
Equity Market Neutral	α	0.28	0.75***	0.63***	1.66***	1.17***	1.38***	0.89***
	α_{ps}	-0.04	0.72***	0.59***	1.53***	1.13***	1.29***	0.83***
	β_{ps}	0.38***	0.03	0.05	0.16	0.05	0.11	0.07
Fund of Funds	α	0.76**	0.27	0.83***	0.57	0.43	1.04	0.59*
	α_{ps}	0.54	0.04	0.64***	-0.06	0.25	0.65	0.33
	β_{ps}	0.26**	0.28***	0.23***	0.75***	0.22**	0.47**	0.32***
Fixed Income Arbitrage	α	0.69***	0.63**	0.75***	0.79***	0.84***	0.88***	0.55*
	α_{ps}	0.59**	0.66**	0.77***	0.75***	0.71**	0.83***	0.42
	β_{ps}	0.12	-0.04	-0.02	0.05	0.15	0.06	0.16*
Global Macro	α	1.05***	0.77**	0.81**	0.55	0.98*	0.78	1.26**
	α_{ps}	0.76*	0.41	0.51	0.24	0.73	0.48	0.85
	β_{ps}	0.35***	0.43***	0.37***	0.37***	0.29*	0.36**	0.49***
Long/Short Equity Hedge	α	0.97*	0.52*	1.01***	1.21**	1.17	0.65	1.00**
	α_{ps}	0.59	0.25	0.72***	0.54	0.49	0.21	0.55
	β_{ps}	0.45***	0.33***	0.34***	0.81***	0.81**	0.52***	0.54***
Managed Futures	α	0.88*	0.51	0.96**	0.40	0.28	0.56	1.04*
	α_{ps}	0.70	0.44	0.87*	0.17	0.07	0.47	0.83
	β_{ps}	0.21	0.08	0.11	0.28	0.25	0.11	0.24
Multi-Strategy	α	0.91***	1.05***	0.96***	2.68***	1.73***	2.58***	2.73***
	α_{ps}	0.79**	0.96***	0.88***	2.69***	1.72**	2.57***	2.63***
	β_{ps}	0.14	0.11	0.09	-0.01	0.02	0.02	0.12

Chapter 3

Heterogeneous Beliefs, Trading Volume, Price Volatility and Liquidity

3.1 Introduction

A pervasive characteristics of financial markets is the presence of heterogeneous beliefs among investors. One simple example of heterogeneous beliefs is the variety of opinions among financial analysts and macroeconomists regarding future movements of earnings per share, interest rates, exchange rate, and gross national product despite the fact that all these analysts have access to the same economic data. It is by now well recognized that heterogeneous beliefs among investors play an important role in the formation of asset prices and their dynamics, and in the generation of trades among investors. Empirical evidence also strongly supports this recognition. Buraschi and Jiltsov (2002) find that differences of opinion can help explain the dynamics of option trading volume while Pavlova and Rigobon (2003) provide empirical support for a model of international stock prices and exchange rates with heterogeneity in beliefs.

There has been a steadily growing literature on models with heterogeneous beliefs among investors regarding some fundamental and non-fundamental aspects of a financial economy. This includes the earlier single or multiple-period discrete-time works of Harris and Kreps (1978), Varian (1985, 1989), Delong et al. (1990), Harris and Raviv (1993), the continuous-time works of Wang (1994), and the subsequent continuous-time developments of Murphy (1994), Zapatero (1998), Basak (2000), Scheinkman and Xiong (2003), Buraschi and Jiltsov (2006). Recently, Vayanos and Vila (2007) and Xiong and Yan (2009) extend this direction research to bond markets.

The economics and finance literature has widely adopted the Bayesian inference framework to model investors' learning processes about unobservable economic variables, such as productivity of the economy and profitability of a specific firm. One line of the literature (e.g., Harris and Raviv (1993), Detemple and Murthy (1994) and Basak (2000)) assumes that investors hold heterogeneous prior beliefs about unobservable economic variables. In these models, investors continue to disagree with each other even after they update their beliefs using identical information, but the difference in their beliefs deterministically converges to zero. In another strand of the literature (e.g., Scheinkman and Xiong (2003), Buraschi and Jiltsov (2006) and Xiong and Yan (2009)), heterogeneous beliefs arise from investors' different prior knowledge about the informativeness of signals and the dynamics of unobservable economic variables, and such heterogeneity, as shown by, for example, Acemoglu, Chernozhukov, and Yildiz (2007), could never converge.

This paper is primarily built upon Kurz and Motolese (2008) which model heterogeneous beliefs using the second approach, and its objective is to study the effects of heterogeneous beliefs on the trading volume, price volatility and liquidity of stocks. Consider an economy where the true probability measure of the sequence of stock payoffs is nonstationary and unobservable. The nonstationarity of economic systems is the source that makes the sequence of stock payoffs to be a nonstationary process which is difficult to be identified by rational investors due to limited data. Historical data can be used to deduce some unique stationary empirical measure, but most investors would not believe that such an empirical measure is adequate to forecast future stock payoffs. Instead, investors hold heterogeneous beliefs when making investment decisions. For each investor, the transition function of his expectation about next period economic state variables is uniquely pinned down by his individual belief. The equilibrium can be solved by treating individual states of belief and its average as state variables.

In equilibrium, stock price is linearly positively correlated with market belief which is defined as the average of individual beliefs. When investors' average belief about future stock payoffs is higher than the econometrician's belief[1], they will value stocks more aggressively and then the equilibrium stock prices would appear "expensive" to the econometrician. This result is similar to that one obtained by Xiong and Yan (2009), which shows that bond prices aggregate investors' heterogeneous beliefs and, in particular, reflects their wealth-weighted average belief about future short rates. A natural result following the average-belief-related stock price is that price volatility is positively correlated with the volatility of market belief: price volatility increases when market belief is more volatile.

Investor's optimal stock *demand* is an increasing function of the difference between his individual belief and market belief: the more optimistic than the market he is, the higher demand he has. *Stock trading volume* is determined by changes in the difference, and its unconditional expectation decreases with the volatility of market belief. In this model, trading volume has nothing to do with belief dispersion among investors, this result is different from previous findings that trading volume increases with the belief dispersion (e.g., Chordia et al. (2007), Xiong and Yan (2009)).

It is also shown in this paper that stock liquidity, measured by the absolute value of stock transaction price minus its fundamental value, decreases with the volatility of market belief. More volatile market belief means higher price pressure and less liquidity given that price pressure is a liquidity measure and higher price pressure means less liquidity.

The theoretically predicted relations between the volatility of market belief and the trading volume, price volatility and liquidity of stocks are empirically checked using the analyst

[1]The econometrician's belief is derived with empirical data using statistical models. Refer to Section 3.2 for the details.

forecast data on quarterly earnings per share provided by the Institutional Brokers' Estimate System, the empirical results support these relations and are robust to methods of estimating market belief and its volatility and to alternative liquidity measure.

This study differs from the existing literature in the following ways: first, it is shown in this paper that market belief and its volatility rather than belief dispersion are the key factors influencing the trading volume and price volatility of stocks. In particular, as mentioned above, trading volume is not affected by belief dispersion in this model; second, this paper also shows that stock liquidity, measured by price pressure, is positively correlated with the volatility of market belief. It seems that no study has formally checked the relation between heterogeneous beliefs and stock liquidity before; third, the objectives of this paper and Kurz and Motolese (2008) are different although they use similar theoretical models.

The rest of this paper is organized as follows: Section 3.2 develops a theoretical model to study how heterogeneous beliefs among investors affect the trading volume, price volatility and liquidity of stocks. Empirical results are presented in Section 3.3. Section 3.4 concludes this paper with further discussions and comments.

3.2 The Model

3.2.1 The Setting

Following Kurz and Motolese (2008), consider an infinite horizon exchange economy where a stock or a portfolio of stocks is traded, of which market price is p_t and which generates an exogenous risky sequence of payoffs $\{D_t, t = 1, 2, \cdots\}$ under a true probability measure $\hat{\Pi}$. For simplicity, assume that the total supply of stock shares is normalized to one.

In this economy, a key assumption is that investors do not know the true $\hat{\Pi}$ of the process $\{D_t, t = 1, 2, \cdots\}$, and they hold heterogeneous beliefs about it. Instead, the historical data on a set of observable variables including D_t is known to all investors, and it plays a role of the common knowledge basis of investors with heterogeneous beliefs. With a long history of observations of the variables, all investors compute the same empirical moments and the same finite dimensional distributions of the observed variables. Using the standard extension of measures they deduce from the data a unique empirical probability measure denoted by $\hat{m}$ on infinite sequences. Kurz (1994) has shown that this measure $\hat{m}$ is stationary. This is the empirical knowledge shared by all investors. Moreover, we assume that the data reveals that, under the probability measure $\hat{m}$, $\{D_t, t = 1, 2, \cdots\}$ constitutes a first order Markov process where D_{t+1} is conditionally normally distributed with mean $\mu + \lambda_d (D_t - \mu)$ and variance

σ_d^2. To simplify, let's define $d_t = D_t - \mu$, the sequence $\{d_t, t = 1, 2, \cdots\}$ is hence zero mean with an unknown true probability Π and an empirical probability m. The assumption that $\{D_t, t = 1, 2, \cdots\}$ constitutes a first order Markov process implies that under the empirical probability measure m the dynamics of $\{d_t, t = 1, 2, \cdots\}$ is also characterized by a first order Markov process with transition function

$$d_{t+1} = \lambda_d d_t + \epsilon_{t+1}^d, \tag{3.1}$$

where $\epsilon_{t+1}^d \overset{m}{\sim} N(0, \sigma_d^2)$. It is clear that $E_t^m(d_{t+1}) = \lambda_d d_t$.

It is well known that our society has undergone changes in firm organization and technology, which are rapid with important economic effects, making $\{d_t, t = 1, 2, \cdots\}$ to be a non-stationary process. This indicates that the distribution of d_t is time dependent, such variability makes it almost impossible to learn the true probability distribution, this is why investors do not know Π.

Assume that there are N investors in the economy, at date t investor i buys θ_t^i shares of stock which will pay him $d_{t+1} + \mu$ at date $t + 1$ for each of θ_t^i. The riskless interest rate $R = 1 + r_t$ is assumed to be constant over time such that there exists a technology by which an investor can invest the amount B_t at date t and receive with certainty the amount $B_t R$ at date $t + 1$. The consumption of investor i at date t is set equal to the income earned from the portfolio $\left(\theta_{t-1}^i, B_{t-1}^i\right)$ held from date $t - 1$ to t minus the cost spent for the portfolio (θ_t^i, B_t^i) created at date t

$$c_t^i = \theta_{t-1}^i (p_t + d_t + \mu) + B_{t-1}^i R - \theta_t^i p_t - B_t^i$$

and his wealth at date t is defined as

$$W_t^i = c_t^i + \theta_t^i p_t + B_t^i$$

and hence the transition formula of wealth takes a form as follows

$$W_t^i = \left(W_{t-1}^i - c_{t-1}^i\right) R + \theta_{t-1}^i Q_t, \tag{3.2}$$

where $Q_t = p_t + (d_t + \mu) - R p_{t-1}$[2]. Given some initial values (θ_0^i, W_0^i) and with an exponential

[2]As Wang (1994), we call Q_{t+1} the excess share return as it is the excess return on one share of stock instead of the excess return on one dollar invested in the stock.

utility function, investor i will choose (θ^i, c^i) to maximize the expected utility

$$U = E_t^i \left[\sum_{s=0}^{\infty} -\beta^{t+s-1} e^{-\frac{1}{\tau} c_{t+s}^i} | \Im_t \right] \tag{3.3}$$

subject to a vector of state variables ψ_t^i and their transition functions which will be specified later. β is a time discount factor and $\Im_t$ is a σ-field generated by the information available up to time t.

3.2.2 The Equilibrium without Heterogeneous Beliefs

Before solving the full model specified in the following sections, let's first consider the case in which investors' beliefs are homogeneous, and they only use the information embedded in the empirical data of d_t to make their investment decisions.

Assume that R=1+r>1 and 0< λ_d <1, the approach suggested by Blanchard and Kahn (1980) can be adopted to solve the equilibrium[3]. The optimal stock demand of each investor and the equilibrium stock price are stated in the following proposition:

PROPOSITION I: *Given the empirical transition equation (1) for d_t and the constrained conditions R=1+r>1 and $0 < \lambda_d < 1$, the optimal stock demand is identical across investors*

$$\theta_t^* = \frac{R\tau}{r\tilde{\sigma}_Q^2} \left[E_t^m \left(Q_{t+1} \right) + u_0^* + u_1^* d_t \right], \tag{3.4}$$

where $[u_0^*, u_1^*]$=$-\tilde{b}\tilde{\Omega}\tilde{\Lambda}_0$, *and* $\tilde{\sigma}_Q^2 = \tilde{b}^2\tilde{\Omega}$ *is the adjusted conditional variance of the excess share return* Q_{t+1}. $\tilde{b}$, $\tilde{\Omega}$ *and* $\tilde{\Lambda}_0$ *are defined as*

$$\tilde{b} = a_d^* + 1,\ \tilde{\Omega} = \sigma_d^2 / \left[1 + \sigma_d^2 \tilde{v}_{11} \right],\ \tilde{\Lambda}_0 = \left[\tilde{v}_{01}, \lambda_d \tilde{v}_{11} \right],\ \tilde{V} = \left[\tilde{v}_{00}\ \tilde{v}_{01}; \tilde{v}_{01}\ \tilde{v}_{11} \right],$$

where $\tilde{V}$ *is determined in the following equation:*

$$\frac{\tilde{M}}{R} - \tilde{V} + 2\left[\gamma\tilde{C} + ln\left(\frac{r}{R} \right) \right] \tilde{\boldsymbol{i}}_{11} = 0,$$

$$\tilde{M} = \frac{1}{\tilde{b}^2\tilde{\Omega}} \left[\tilde{a}^T - \tilde{b}\tilde{\Omega}\tilde{\Lambda}_0 \right]^T \left[\tilde{a}^T - \tilde{b}\tilde{\Omega}\tilde{\Lambda}_0 \right] + \left[\tilde{\Lambda}_\psi^T \tilde{V} \tilde{\Lambda}_\psi - \tilde{\Lambda}_0^T \tilde{\Omega} \tilde{\Lambda}_0 \right],$$

where $\tilde{C}$=$\frac{-ln(r\beta\tilde{G})}{\gamma R}$, $\tilde{a}$=$[P_0^* (1-R) + \mu, (a_d^* + 1)\lambda_d - a_d^* R]^T$, $\tilde{\Lambda}_\psi$=$[1\ 0; 0\ \lambda_d]$, $\tilde{G} = (1+\sigma_d^2\tilde{v}_{11})^{-\frac{1}{2}}$

[3]To ensure the existence of the equilibrium of a linear difference model, it is required that the exogenous variable, here d_t, does not *exponentially grow*, d_t satisfies this condition when $0 < \lambda_d < 1$.

and $\tilde{\boldsymbol{\imath}}_{11}$ *is a* 2×2 *matrix with the element* $(1,1)$ *being one and other elements being zero. Given that the total supply of stock shares is normalized to one, the equilibrium stock price is unique and takes a form as follows:*

$$p_t^* = P_0^* + a_d^* d_t, \tag{3.5}$$

where the constant term P_0^* *and the coefficients* a_d^* *are defined as*

$$P_0^* = \frac{\mu + u_0^*}{R-1} - \frac{\tilde{\sigma}_Q^2}{RN\tau}, \quad a_d^* = \frac{\lambda_d + u_1^*}{R - \lambda_d}$$

Proof: This proposition can be proved as a special case in the Appendix in Section 3.5.1. ||

It is evident that when investors' beliefs are homogeneous, the optimal demand of stock is identical across investors as it is independent of investors' individual beliefs. For each investor, his demand can be divided into two parts: one is the investment demand driven by the expected excess share return $E_t^m(Q_{t+1})$ and another is driven through hedging against the fluctuations of d_t, the coefficients u_0^* and u_1^* determine the magnitude of hedging demands. It is shown in Figure 1 that the values of u_0^* and u_1^* for relevant values of the model parameters are very close to zero and then almost negligible. To insert the equilibrium stock price p_t^* (3.5) into the demand function (3.4) and to simplify lead to $\theta_t^* = 1/N$, that is, over time, when the beliefs of investors are homogeneous, each investor will constantly demand 1/N of total stock supply and no trading will occur among investors except for the first period when the initial stock endowments of investors may be different from 1/N.

The equilibrium price p_t^* derived in this case consists of two components: the first one is P_0^* which can be further divided into two parts: the first part $\frac{\mu}{r}$ is the present value of expected total future cash flows discounted at the risk free interest rate, the second part, approximated by the term -$\frac{\tilde{\sigma}_Q^2}{RN\tau}$ as u_0^* is close to zero, represents the discount on the stock price to compensate for the risk in its future cash flows and the discount increases with the adjusted conditional variance $\tilde{\sigma}_Q^2$ of Q_{t+1} and decreases with the risk tolerance τ of investors and the number of investors N, this result is consistent with our intuition; the second component of the equilibrium price depends on d_t. The effect of d_t on the equilibrium price p_t^* is determined by the coefficients a_d^*. As we see above, the coefficient u_1^* is almost equal to zero, therefore the coefficient a_d^* is always strictly positive as shown in Figure 1 and then p_t^* increases with d_t.

3.2.3 Modeling Heterogeneous Beliefs

Apparently, most investors don't believe that the empirical stationary model (3.1) is adequate to forecast the future as $\{d_t, t = 1, 2, \cdots\}$ is non-stationary with unknown probability Π. Each investor holds his personal belief about d_{t+1}. Define that the belief of an investor is rational if his subjective model is consistent with the data and if simulated, its simulated data reproduces the stationary probability m deduced from historical data. All investors are assumed to be rational in this economy[4].

In this model, individual beliefs are treated as state variables, and an individual belief about an economic state variable is described as a personal state of belief which can uniquely pin down the transition function of this investor's belief about next period economic state variables. A given personal state of belief does therefore identify the type of an investor. Note that unlike in asymmetric information models, investors are willing to reveal their forecasts in this model, however, other investors don't view such forecasts as a source of new information and will not update their beliefs. As individual beliefs are publicly available, the distribution of individual states of belief is then an economy-wide observable state variable.

Investor i's state of belief is defined as g_t^i which is characterized by the following dynamics

$$g_{t+1}^i = \lambda_Z g_t^i + \epsilon_{t+1}^{ig}, \tag{3.6}$$

where $\epsilon_{t+1}^{ig} \sim N\left(0, \sigma_g^2\right)$ are correlated across i reflecting the correlation of individual beliefs. The unconditional expected value of g_t^i is zero. Assume that investor i knows his own g_t^i and the market distribution of g_τ^l across l for all time $\tau \le t$. Investor i's state of belief g_t^i pins down his own perception of date $t+1$ payoff d_{t+1}^i in the following way

$$d_{t+1}^i = \lambda_d d_t + \lambda_d^g g_t^i + \epsilon_{t+1}^{id} \tag{3.7}$$

where $\epsilon_{t+1}^{id} \sim N\left(0, \hat{\sigma}_d^2\right)$ and $\hat{\sigma}_d^2$ is the same for all investors. Obviously, we have that

$$E^i\left[d_{t+1}^i | \Im_t, g_t^i\right] - E^m\left[d_{t+1} | \Im_t\right] = \lambda_d^g g_t^i \tag{3.8}$$

where the first and second terms in the left side are respectively the investor i's forecast of d_{t+1} and the forecast made under the empirical probability distribution.

Average (3.6) over investors and denote by Z_t the mean of the cross-sectional distribution

[4]The theory of Rational Beliefs (RB in short) used in this paper is due to Kurz(1994, 1997) and Kurz and Motolese(2008), and Kurz and Motolese(2008) has listed all rationality conditions imposed in this model. For simplicity we do not discuss these conditions.

of g_t^i and refer to it as *the market state of belief (or market belief in brief).* The dynamics of Z_t is given by

$$Z_{t+1} = \lambda_Z Z_t + \epsilon_{t+1}^Z \tag{3.9}$$

with $\epsilon_{t+1}^Z \overset{m}{\sim} N(0, \sigma_Z^2)$. Assume that the correlation of individual beliefs is non-stationary and the true distribution of ϵ_{t+1}^Z is unknown, then the process $\{Z_t, t = 1, 2, \cdots\}$ is non-stationary as well. Since both d_t and Z_t are observable, so does the joint empirical distribution of these two state variables. Assume that this joint distribution is described by the following system of equations

$$d_{t+1} = \lambda_d d_t + \epsilon_{t+1}^d \tag{3.10}$$

$$Z_{t+1} = \lambda_Z Z_t + \epsilon_{t+1}^Z \tag{3.11}$$

where the error terms ϵ_{t+1}^d and ϵ_{t+1}^Z are jointly normally distributed as follows

$$\begin{pmatrix} \epsilon_{t+1}^d \\ \epsilon_{t+1}^Z \end{pmatrix} \overset{m}{\sim} N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \sigma_d^2 & 0 \\ 0 & \sigma_z^2 \end{bmatrix}\right) \tag{3.12}$$

Again, most investors do not believe that the stationary models (3.10) and (3.11) are the truth, and they form their personal beliefs. Similar as before, investor i's state of belief g_t^i pins down his perception of the two state variables $\left(d_{t+1}^i, Z_{t+1}^i\right)$ and his belief takes the following form

$$d_{t+1}^i = \lambda_d d_t + \lambda_d^g g_t^i + \epsilon_{t+1}^{id} \tag{3.13}$$

$$Z_{t+1}^i = \lambda_Z Z_t + \lambda_Z^g g_t^i + \epsilon_{t+1}^{iZ} \tag{3.14}$$

$$g_{t+1}^i = \lambda_Z g_t^i + \epsilon_{t+1}^{ig} \tag{3.15}$$

where the error terms ϵ_{t+1}^{id}, ϵ_{t+1}^{iZ} and ϵ_{t+1}^{ig} are jointly normally distributed as follows

$$\begin{pmatrix} \epsilon_{t+1}^{id} \\ \epsilon_{t+1}^{iZ} \\ \epsilon_{t+1}^{ig} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \hat{\sigma}_d^2 & \hat{\sigma}_{dZ} & 0 \\ \hat{\sigma}_{dZ} & \hat{\sigma}_Z^2 & 0 \\ 0 & 0 & \sigma_g^2 \end{bmatrix}\right) \tag{3.16}$$

The stochastic transition functions (3.13)-(3.15) together with the wealth transition process (3.2) constitute the constrained conditions of investor i's optimization problem (3.3).

3.2.4 The Equilibrium with Heterogeneous Beliefs

In this section, we will solve the equilibrium with heterogeneous beliefs. When investors hold heterogeneous beliefs, each investor maximizes his optimal consumption-investment choice based on his individual belief about (d_t, Z_t), and hence the stock demand of investors can be different from each other since their beliefs may be different.

We use the perception models (3.13)-(3.15) about the state variables (d_t,Z_t,g_t), average them over investors and use the definition of Z_t to deduce the following relations:

$$\overline{E}_t\left(d_{t+1}\right) = \lambda_d d_t + \lambda_d^g Z_t \tag{3.17}$$

$$\overline{E}_t\left(Z_{t+1}\right) = \left(\lambda_Z + \lambda_Z^g\right) Z_t \tag{3.18}$$

where $\overline{E}_t(\bullet)$ is an average market expectation operator. Besides the constrained conditions $R > 1$ and $0 < \lambda_d < 1$ as in the case without heterogeneous beliefs, it is also assumed that $0 < \lambda_Z + \lambda_Z^g < 1$ to make sure that Z_t is on average stationary.

PROPOSITION II: *Given the perception equations (3.13)-(3.15), the relations (3.17)-(3.18) and the constrained conditions R>1,* $0 < \lambda_d < 1$ *and* $0 < \lambda_Z + \lambda_Z^g < 1$*, the optimal stock demand of investor i is:*

$$\theta_t^i = \frac{R\tau}{r\hat{\sigma}_Q^2}\left[E_t^i\left(Q_{t+1}\right) + u_0 + u_1 d_t + u_2 Z_t + u_3 g_t^i\right] \tag{3.19}$$

where $\hat{\sigma}_Q^2$ *is the adjusted conditional variance of the excess share return* Q_{t+1}*.Given that the total number of outstanding shares is normalized to one, the equilibrium stock price is unique and takes a form as follows*

$$p_t = P_0 + a_d d_t + a_Z Z_t, \tag{3.20}$$

where the constant term P_0*, the coefficients* a_d *and* a_Z *are defined as*

$$P_0 = \frac{\mu + u_0}{R-1} - \frac{\hat{\sigma}_Q^2}{RN\tau},\ a_d = \frac{\lambda_d + u_1}{R - \lambda_d},\ a_Z = \frac{\left(a_d + 1\right)\lambda_d^g + u_2 + u_3}{R - \left(\lambda_Z + \lambda_Z^g\right)}.$$

Proof: See the Appendix in Section 2.5.1. $\|$

Note that the results in this proposition are similar to the previous results derived without heterogeneous beliefs except that in the full model, besides the investment demand associated with $E_t^i\left(Q_{t+1}\right)$ and the hedging demand against the fluctuations of d_t, the optimal stock demand of investor i is also linked to market belief Z_t and his own personal belief g_t^i which are both subjective, different beliefs will lead to different demands. The equilibrium stock price also depends on market belief Z_t, and how Z_t affects p_t is determined by the coefficient a_Z. There are no analytical solutions for the parameters $u = [u_0, u_1, u_2, u_3]$ and then a_d and a_Z, we use the Monte-Carlo method to numerically compute these parameters for relevant values of the model parameters, and the results are shown in Figure 2. This figure shows that a_Z is strictly positive for relevant values of the model parameters, it can be thus concluded that p_t increases with market belief Z_t.

3.2.5 Trading Volume

Inserting the price formula (3.20) into the demand function of investor i (3.19) and to simplify, we obtain

$$\theta_t^i = \frac{1}{N} + \frac{R\tau}{r\hat{\sigma}_Q^2}\left[\left(a_d + 1\right)\lambda_d^g + a_Z\lambda_Z^g + u_3\right]\left(g_t^i - Z_t\right) \tag{3.21}$$

The optimal demand of investor i in the equilibrium with heterogeneous beliefs is not equal to 1/N any more, indeed it depends on the difference between his individual belief g_t^i and market belief Z_t. As shown in Figure 2, the numerically calculated $\left(a_d + 1\right)\lambda_d^g + a_Z\lambda_Z^g + u_3$ is strictly positive for relevant values of the model parameters, this means that when investor i is more optimistic than the market, his demand will exceed 1/N, and vice versa. However if beliefs are identical across investors, then, same as in the case without heterogeneous beliefs, the demand of each investor is 1/N. Trading will take place among investors when the differences between individual beliefs and market belief change over time. Note that the concept of 'trading' here is different from the demand, high demand does not mean actual buying in, and vice versa. Since the total number of stock shares outstanding is normalized to one, what refers to as trading volume is actually *turnover rate*. For investor i, the stock volume he will trade at time t, denoted by V_t^i, is

$$V_t^i = |\theta_t^i - \theta_{t-1}^i| = \overline{C}_V\left|\left(g_t^i - g_{t-1}^i\right) - \left(Z_t - Z_{t-1}\right)\right|, \tag{3.22}$$

where $\overline{C}_V = \frac{R\tau}{r\hat{\sigma}_Q^2}|\left(a_d + 1\right)\lambda_d^g + a_Z\lambda_Z^g + u_3|$, and the aggregate trading volume denoted by V_t is

$$V_t = \frac{1}{2}\sum_{i=1}^{N} V_t^i \tag{3.23}$$

The sum of V_t^i is divided by two because buying and selling can be only accounted once. Note that, in this model, higher belief dispersion doesn't mean larger trading volume. This result is different from previous findings by, for example, Chordia et al. (2007), Xiong and Yan (2009). For investor i, we have that

$$\overline{V}^i = E\left(V_t^i\right) = \overline{C}_V\sqrt{\frac{2}{\pi}}\sigma_V, \tag{3.24}$$

$$var\left(V_t^i\right) = \left(\frac{\pi}{2} - 1\right)\left(\overline{V}^i\right)^2, \tag{3.25}$$

where $\sigma_V^2 = \frac{2}{(1+\lambda_Z)\left(1-\lambda_Z^2\right)}\left[(1+\lambda_Z+\lambda_Z^g)(1-\lambda_Z) - \lambda_Z\left(\lambda_Z^g\right)^2\right]\sigma_g^2 + \frac{2}{1+\lambda_Z}\sigma_Z^2$, and it is identical across investors. The variance of trading volume thus increases quadratically with its mean. The aggregate expected trading volume in the market, which is denoted by $\overline{V}$, is given by:

$$\overline{V} = \frac{1}{2}\sum_{i=1}^{N}\overline{V}^i = \frac{1}{2}N\overline{C}_V\sqrt{\frac{2}{\pi}}\sigma_V \tag{3.26}$$

Figure 3 plots the numerically computed aggregate expected trading volume $\overline{V}$ against the variance of market belief σ_Z^2 for relevant values of the model parameters. Increasing σ_Z^2 has two opposite effects on $\overline{V}$: σ_V increases while $\overline{C}_V$ decreases. That $\overline{C}_V$ decreases with σ_Z^2 is due to the fact that the adjusted variance of excess share return $\hat{\sigma}_Q^2$ increases with σ_Z^2 while $(a_d+1)\lambda_d^g + a_Z\lambda_Z^g + u_3$ decreases with σ_Z^2 as shown in Figure 2B. The net effect is that $\overline{V}$ decreases with σ_Z^2, that is, the expected trading volume declines when market belief is more volatile.

3.2.6 Price Volatility

There exist two types of price variability: one is the instantaneous variance and another is the unconditional variance (i.e. price volatility). The instantaneous variance is important in analyzing trading strategies while price volatility is often adopted in empirical testing. In the following, I will only show the results for price volatility as the results for the instantaneous variance are just similar.

First, consider the price volatility for the case in which individual beliefs are homogeneous. Given the price process (3.5), price volatility, denoted by $\sigma_{p^*}^2$, is

$$\sigma_{p^*}^2 = Var\left(p_t^*\right) = \frac{2a_d^{*2}}{1+\lambda_d}\sigma_d^2 \tag{3.27}$$

Figure 4A plots the numerically simulated price volatilities for relevant values of the model parameters. Clearly, stock price tends to become more volatile when the variance of d_t under the empirical probability m increases. Note that the relation between $\sigma^2_{p^*}$ and σ^2_d looks like linear, this is because the effect of σ^2_d on a^*_d is infinitesimal, and for any relevant values of the model parameters, a^*_d almost doesn't change with σ^2_d. In the case without heterogeneous beliefs, price volatility is uniquely determined by the innovations to d_t.

However, in the real financial markets, stock price is more volatile. To see this point, we calculate the volatility of market price p_t (denoted by σ^2_p) under the probability distribution m as follows:

$$\sigma^2_p = Var\left(p_t\right) = \frac{2a^2_d}{1+\lambda_d}\sigma^2_d + \frac{2a^2_Z}{1+\lambda_Z}\sigma^2_Z \tag{3.28}$$

Therefore, the volatility of market price is attributed to not only the innovations to d_t, but also the innovations to market belief Z_t. The numerically computed σ^2_p is plotted in Figure 4B. The first obvious result to be found in this figure is that the volatility of market price increases with both σ^2_d and σ^2_Z. Second, σ^2_Z has a larger effect on σ^2_p than σ^2_d while the effects of both σ^2_Z and σ^2_d are nonlinear due to the nonlinear effect of σ^2_d and σ^2_Z on a_Z as shown in Figure 2B. The third and also most important result is that for any given value of σ^2_d, market price could be much more volatile than the hypothetical price given in equation (3.5) when market belief Z_t is reasonably sufficiently volatile, this can be found out by comparing the results in Figure 4A and Figure 4B.

3.2.7 Liquidity

As Kyle (1985) argues, *liquidity* is a slippery and elusive concept, in part because it encompasses a number of transactional properties of financial markets. These properties include *tightness* (the cost of turning around a position over a short time period), *depth* (the size of an order flow innovation required to change prices a given amount), and *resiliency* (the speed with which prices recover from a random, uninformative shock). The liquidity measure used in this paper is closely related to its first property-*tightness*[5]. Define Λ_t as follows:

$$\Lambda_t = p_t - p^*_t, \tag{3.29}$$

which is the (signed) deviation of the realized (or market) price (3.20) from the hypothetical price (3.5) implied by the empirical data, and our measure of stock liquidity is defined as the

[5]There does not exist any measure which is able to capture all the three types of liquidity properties.

absolute value of this deviation $|\Lambda_t|$[6]. In the financial markets, at any time point, the real value of a stock is unobservable, the price p_t^* can be however regarded as an over time average of the real values of the stock provided that the empirical probability m is indeed an average over an infinite sequence of true probability $\hat{m}$ regimes, reflecting long term frequencies. The liquidity measure in this paper is similar to that one adopted by Brunnermeier and Pedersen (2009). $|\Lambda_t|$ measures price pressure or transitory price effect which is the essence of liquidity according to Grossman and Miller (1988), and Hendershott and Menkveld (2009).

Given the price formulas (3.5) and (3.20), we have

$$|\Lambda_t| = |\Delta P_0 + \Delta a_d d_t + a_Z Z_t| \tag{3.30}$$

where $\Delta P_0 = P_0 - P_0^*$ and $\Delta a_d = a_d - a_d^*$. Apparently, the liquidity $|\Lambda_t|$ of stock is a function of the market state of belief Z_t. Setting $|\Lambda_t| = 0$, that is $p_t = p_t^*$, leads to

$$Z_t^E = -\frac{\Delta P_0}{a_Z} - \frac{\Delta a_d}{a_Z} d_t \tag{3.31}$$

The liquidity problem will arise when market belief Z_t deviates from Z_t^E. Given the formula (3.30) and the joint normality assumption about d_t and Z_t, we have

$$\mu_{|\Lambda|} = E^m\left(|\Lambda_t|\right) = \sigma_\Lambda \sqrt{\frac{2}{\pi}} exp\left(-\frac{\mu_\Lambda^2}{2\sigma_\Lambda^2}\right) + \mu_\Lambda \left(1 - 2\Phi\left(-\frac{\mu_\Lambda}{\sigma_\Lambda}\right)\right), \tag{3.32}$$

and the unconditional variance of $|\Lambda_t|$ is

$$\sigma_{|\Lambda|}^2 = var^m\left(|\Lambda_t|\right) = \mu_\Lambda^2 + \sigma_\Lambda^2 - \left(\mu_{|\Lambda|}\right)^2, \tag{3.33}$$

where $\mu_\Lambda = \Delta P_0$ and $\sigma_\Lambda^2 = (\Delta a_d)^2 \frac{\sigma_d^2}{1-\lambda_d^2} + a_Z^2 \frac{\sigma_Z^2}{1-\lambda_Z^2}$. The proof can be found in the Appendix in Section 3.5.2. As plotted in Figure 5, the expected value of $|\Lambda_t|$ increases with σ_Z^2, this means that stock is expected to become illiquid when market belief is more volatile. This result, together with the results in Section 3.2.5, allows us to conclude that the higher σ_Z^2 is associated with less trading volume and illiquidity. The phenomenon of the coexistence of less trading volume and illiquidity has been documented in the financial markets, the model in this article provides an alternative explanation for this phenomenon.

Different from the inventory risk and asymmetric information theories, it is shown in this model that even when there is no inventory risk or information asymmetry, liquidity can be still a relevant problem in the financial markets as a result of heterogeneous beliefs among

[6] Indeed, $|\Lambda_t|$ measures illiquidity, but for simplicity, we still call it liquidity measure.

investors. To our knowledge, this paper is the first one to demonstrate such relation between heterogeneous beliefs and stock liquidity.

3.3 Empirical Analysis

In this section, the theoretically predicted relations between the volatility of market belief about stock payoffs and the trading volume, price volatility and liquidity of underlying stocks derived in Section 3.2 will be empirically checked using the analyst forecast data provided by the Institutional Brokers' Estimate System (I/B/E/S).

The mission before selecting data is to choose a poxy variable for stock payoffs, of which analyst forecast data should be available. There are two candidates for such proxy variable: one is dividend per share (DPS) and another is earnings per share (EPS), and both of them are used by financial researchers. DPS is however affected very much by the dividend policy which is difficult to control for when we make empirical tests, more importantly, the analyst forecast data on DPS only has short history and the coverage of analysts for DPS forecast is rather low. Due to these reasons, EPS will be used as a proxy variable for stock payoffs in this paper.

With the actual and forecast data on EPS provided by the I/B/E/S, time-series models are adopted to predict EPS, and the market state of belief about EPS can be constructed from the predicted EPS. The volatility of market belief is estimated with both rolling regression method and GARCH model, and finally empirical analysis will be done using the estimated volatility of market belief.

3.3.1 Data

The empirical works in this paper focus on the S&P500 index stocks, this is mainly due to the fact that smaller firms are covered with much fewer financial analysts so that analysts' earnings forecasts don't fully represent the views of most investors and hence market belief estimated from such earnings forecasts may be biased.

The analyst forecast data on EPS are from the I/B/E/S U.S. Summary History database which contains summary statistics for analysts' earnings forecasts, including forecast mean, median, standard deviation and the number of financial analysts. These data are in general disclosed on the third Tuesday of each month. Each record also contains the revision date on which the forecast was last confirmed to be accurate.

The actual EPS data are also taken from the I/B/E/S, and it is called the 'Street' EPS since it is tracked by analysts and priced by investors. The COMPUSTAT also provides EPS data which is known as the GAAP EPS that is reported in firms' financial statements. But, as documented by Bradshaw et al. (2000), there exists a large and growing gap between the 'Street' EPS and the GAAP EPS since the former excludes cost items such as 'non-recurring' and 'no-cash' charges. Since analysts' EPS forecasts are based on the 'Street' defined EPS, it makes sense for us to use the 'Street' EPS data rather than the GAAP EPS data to predict EPS that will be used to construct market belief although the GAAP EPS data seems more transparent.

The EPS data (both actuals and forecasts) collected by the I/B/E/S have different periodicities: quarterly, semi-annually, annually, etc. In this paper, we choose to use quarterly EPS data because of two reasons: first, the coverage of analysts is relatively high for quarterly EPS forecast; second, in the accounting literature, most time series models have been developed to predict quarterly EPS.

The stocks included in the sample must meet two criteria: 1) Quarterly continuous actual EPS data for the 1983-2008 period; 2) Monthly continuous forecast mean data on EPS for the 1993-2008 period. 114 S&P500 index stocks meet these two criteria, and Table 1 details the industry composition of the sample along with GICS two-digit industry codes, stocks from three industry sectors: 'Industrials', 'Consumption Discretionary', and 'Consumption Staples', comprise more than 50% percent of the sample while there are just few stocks from 'Telecommunication Services' and 'Utilities' sectors.

Common stock data are obtained from the CRSP database.

3.3.2 Time-Series Models

To construct market belief Z_t, we need to first predict quarterly EPS. One simple time-series model which can do the prediction is the seasonal random walk with drift model which can be written as follows:

$$E\left(Q_t\right) = \delta + Q_{t-4} \tag{3.34}$$

where $E\left(Q_t\right)$ is the earnings forecast for quarter t, δ is a (typically positive) trend term, and Q_{t-4} is the actual earning for quarter $t-4$. The advantage of this model is that it can capture the seasonality characteristics in the quarterly earnings data documented by, for example, Lorek (1979) among others. However, such a model is not very accurate, in general, because seasonal differences in quarterly earnings, $Q_t - Q_{t-4}$, are affected by factors other than a trend term. In particular, seasonal differences for quarter t typically exhibit diminishing positive correlation with the three prior quarters' seasonal differences ($Q_{t-1} - Q_{t-5}$, $Q_{t-2} - Q_{t-6}$, $Q_{t-3} - Q_{t-7}$) and a negative correlation with the seasonal difference four quarters prior ($Q_{t-4} - Q_{t-8}$). Brown and Rozeff (1979) find that the following model, which incorporates this autocorrelation structure, most accurately predicts the quarterly earnings per share:

$$E\left(Q_t\right) = \delta + Q_{t-4} + \phi\left(Q_{t-1} - Q_{t-5}\right) + \theta\epsilon_{t-4} \tag{3.35}$$

where Q_{t-k} is the actual earnings for quarter $t-k$, ϵ_{t-4} is the earnings shock experienced at quarter $t-4$, and, in general, $\phi > 0$ and $\theta < 0$. This model contains an autoregressive component: $Q_{t-1} - Q_{t-5}$ which reflects the positive autocorrelations in seasonal quarterly differences at the first three lags and a moving average component ϵ_{t-4} which is responsible for the negative correlation in seasonal difference at the fourth lag. In this paper, both these two time-series models will be used to predict quarterly EPS while the first one is for the purpose of robustness test.

For each stock, the prediction of EPS for each quarter over the sample period between 1993 and 2008 is derived using the estimated coefficients from either a regression of Q_t on Q_{t-4} or a regression of seasonal changes in actual EPS for quarter t, $Q_t - Q_{t-4}$, on $Q_{t-1} - Q_{t-5}$ and ϵ_{t-4} (depending on which time-series model is used), and each regression is based on 40 quarters of historical EPS. A time-series of 64 estimates for each coefficient in both the seasonal random walk with drift model and the Brown and Rozeff (1979) model are obtained for each stock, the cross-sectional statistics of time series means of the estimated coefficients is reported in Table 2. Consistent with the previous findings in the literature, the estimated coefficients $\hat{\delta}$ and $\hat{\phi}$ are positive and the estimated coefficient $\hat{\theta}$ is negative in general.

3.3.3 Market State of Belief

Denote by $E_t^i(eps_{t+1})$ investor $i's$ conditional forecast of quarter $t+1$ EPS and by $E_t^m(eps_{t+1})$ the prediction under the stationary empirical probability m. Investor $i's$ state of belief about EPS is defined as[7]:

$$g_t^{eps,i} = E_t^i\left(eps_{t+1}\right) - E_t^m\left(eps_{t+1}\right) \tag{3.36}$$

[7]This definition is inspired by Eq. (3.8)

A positive $g_t^{eps,i}$ means that investor i is optimistic about the earnings ar quarter $t+1$. Market belief is defined as the average of individual beliefs across investors:

$$Z_t^{eps} = \frac{1}{N}\sum_{i=1}^{N}\left[E_t^i\left(eps_{t+1}\right) - E_t^m\left(eps_{t+1}\right)\right] = \overline{E}_t\left(eps_{t+1}\right) - E_t^m\left(eps_{t+1}\right) \tag{3.37}$$

where $\overline{E}_t\left(eps_{t+1}\right)$ is the average forecast across investors, and Z_t^{eps} reflects the market's views about the earnings at quarter $t+1$.

To construct Z_t^{eps}, we need data on both $\overline{E}_t\left(eps_{t+1}\right)$ and $E_t^m\left(eps_{t+1}\right)$. The forecast mean of quarterly EPS provided by the I/B/E/S can be used as a proxy for the average forecast $\overline{E}_t\left(eps_{t+1}\right)$, and as for $E_t^m\left(eps_{t+1}\right)$, it can be predicted using the time-series models proposed in Section 3.3.2. It is worthful to notice that the frequencies of $\overline{E}_t\left(eps_{t+1}\right)$ and $E_t^m\left(eps_{t+1}\right)$ are different: the former is in month but the latter is in quarter. When constructing monthly market belief, $E_t^m\left(eps_{t+1}\right)$ needs to be subtracted from all monthly forecast means for quarter $t+1$.

Figure 7 plots the graphs of market beliefs over the 1993-2008 period for all sample stocks while the upper graph traces market beliefs estimated with the Brown and Rozeff (1979) model and the bottom graph traces market beliefs estimated with the seasonal random walk with drift model, this figure shows that market beliefs fluctuate dramatically over time.

Table 3 reports the cross-sectional statistics of time-series means, skewness, and kurtosis of market beliefs. It is clear that the mean of market beliefs is not zero over short time period although the theory requires to have a long-term time average equal to zero, the market is on average optimistic about the future earnings. Moreover, market beliefs are distributed with heavy-tail, indicating that market beliefs can take extreme values.

3.3.4 Volatility of Market Belief

The volatility of market belief is unobservable, but there are different methods to estimate volatility in the finance literature. In this paper, as a benchmark case, we use rolling regression volatility estimators with window length τ for each stock j, namely,

$$\hat{var}_t\left(Z_{j,t}^{eps}\right) = \sum_{k=1}^{\tau}\omega_k\left(Z_{j,t+1-k}^{eps} - \mu_{j,t}\right)^2 \tag{3.38}$$

where $\mu_{j,t} = \sum_{k=1}^{\tau}\omega_k Z_{j,t+1-k}^{eps}$, the weights ω_k decline geometrically with $\sum_{k=1}^{\tau}\omega_k = 1$, and τ represents the window length. A type of geometrically declining weights $\omega_k = e^{-\alpha k}$ proposed by Foster and Nelson (1996) is used in this paper, and the window length τ is randomly set equal to four[8]. The volatility of market belief could be estimated using the squared periodic market states of belief, however, since volatilities are in general persistent, as we have learned from the stochastic volatility literature, such an estimator of volatility will be then biased and inefficient, rolling regression estimators can somehow avoid this problem. Another advantage of rolling regression estimators is that they don't need intensive computations. In addition to rolling regression method, as a robustness check, we also use a GARCH(p, q) model with orders $p = q = 1$ to estimate the volatility of market belief, which takes a form as follows[9]:

$$Z_t = \phi_0 + \phi_1 Z_{t-1} + \epsilon_{z,t} \tag{3.39}$$

$$\epsilon_{z,t} = \sigma_{z,t} v_{z,t} \tag{3.40}$$

where $\{v_{z,t}\}$ is a *i.i.d* $N(0, 1)$, and $\{\sigma_{z,t}\}$ satisfies the recurrence equation

$$\sigma^2_{z,t} = \alpha_0 + \alpha_1 \epsilon^2_{z,t-1} + \beta_1 \sigma^2_{z,t-1} \tag{3.41}$$

Table 4 details the cross-sectional statistics of time series means of the estimated volatility of market belief. It is shown in this table that the volatility of market belief estimated with the GARCH(1,1) model is larger. Adding longer lagged market beliefs (i.e., Z_{t-2}, Z_{t-3}, ...) into Eq. (3.41) can reduce the estimated volatility of market belief, but this doesn't change the conclusions made later in this paper. The magnitude of the volatility of market belief varies dramatically among stocks, as indicated by the fact that the cross-sectional standard deviation of the volatility of market belief is much larger than its mean, it seems reasonable that the market states of belief for small and growing firms may be more volatile.

3.3.5 Trading Volume, Price Volatility and Liquidity

The data which will be used to compute the variables of *trading volume*, *price volatility* and *liquidity measures* are taken from the CRSP database.

• *Trading Volume*: in this paper, *stock turnover rate* will be used as the measure of trading volume, which is defined, for each stock, as the ratio of number of shares traded in a month by number of shares outstanding, this definition is consistent with that one in Section 3.2;

• *Price Volatility*: defined as the variance of stock prices in a month;

[8]The results for $\tau = 2$ *or* 6 are similar to what we have obtained with $\tau = 4$, but the results with longer τ are less significant, this may be caused by the possibility that the persistence of the volatility of market belief decline fast so that the volatility estimated with long time prior data are very noise.

[9]The results with other reasonable orders p and q are similar

•*Liquidity*: many liquidity measures have been developed in the literature, such as bid-ask spread, effective bid-ask spread, depth, Kyle's (1985) lambda, etc, each of them reflecting a different aspect of liquidity properties. As explained in Section 3.2.7, the liquidity measure used in the theoretical model is price pressure. Among existing liquidity measures, effective bid-ask spread is a good proxy for price pressure. In the following empirical tests, we will use the effective bid-ask spread developed by Roll (1984) as liquidity measure, and this effective spread can be estimated in the following way: at time t, denote by p_t transaction price for a trade, which may be expressed as

$$p_t = m_t + q_t c \tag{3.42}$$

$$m_t = m_{t-1} + u_t \tag{3.43}$$

where m_t denotes the efficient price and u_t are i.i.d zero mean random variables with variance σ_u^2, q_t is a trade direction indicator set to +1 if the customer is buying and -1 if the customer is selling, and c is transaction cost which is not related to the dynamics of m_t. It can be seen from Eq. (3.42) that c measures the deviation of transaction price p_t from the efficient price m_t and is a type of price pressure. Assume that buys and sells are equally likely, serially independent, and that investors buy or sell independently of u_t. Denote $\Delta p_t = p_t - p_{t-1}$, it is easy to show that $E(\Delta p_t) = 0$. The first order autocovariance of price changes is

$$\begin{aligned}\gamma_1 &= cov\left(\Delta p_{t-1},\ \Delta p_t\right) = E\left(\Delta p_{t-1}\Delta p_t\right) \\ &= E\left[c^2(q_{t-2}q_{t-1} - q_{t-1}^2 - q_{t-2}q_t + q_{t-1}q_t) + c(u_{t-1}q_t - u_{t-1}q_{t-1} + q_{t-1}u_t - q_{t-2}u_t)\right] \\ &= -c^2 \end{aligned} \tag{3.44}$$

We obtain that $c = \sqrt{-\gamma_1}$. The first order autocovariance γ_1 is not always negative, c will be set to zero in case that $\gamma_1 > 0$. For each stock, c is assumed to be constant within a given month and is estimated using daily price data in that month;

In every month, to compute trading volume, price volatility and effective spread, stocks are required to at least have 15 daily transaction data in that month, otherwise the values of these variables are set to zero.

The columns 2–4 of Table 5 report the cross-sectional statistics of time-series means of trading volume, price volatility, and effective spread. The monthly number of traded shares on average represents around 10% of total outstanding shares, and the Roll's effective bid-ask spread is \$0.232 per share. The cross-sectional average of price volatilities is surprising high, it is likely driven by the extreme price volatilities of some stocks, which can be seen in Figure 8. The graphs of time-series trading volumes in Figure 8 show that turnover rate has been increasing over the sample period, this is consistent with the findings by other studies such as Lo and Wang (2001).

3.3.6 Regression Results

3.3.6.1 Univariate Tests

To begin the study of the relations between the volatility of market belief about stock payoffs and the trading volume, price volatility and liquidity of underlying stocks, I first report the results of simple time-series regressions which can be written as follows:

$$Y_{i,t} = \alpha_i + \beta_i vZ_{i,t} + \epsilon_{i,t} \quad (i = 1, ..., 114) \tag{3.45}$$

where Y_t is the value of dependent variable which is either trading volume or price volatility or effective spread at month t. vZ_t is the volatility of market belief, which is estimated using rolling regression method as specified in Section 3.3.4.

Table 6 reports the cross sectional statistics of estimated coefficients $\hat{\beta}_i$. The empirical results are in general consistent with the theoretical predictions derived in Section 3.2. The results for the case in which market belief Z_t is constructed using the Brown and Rozeff (1979) model are reported in the second column of Table 6.

As shown in Panel B, for most of sample stocks, price volatility is significantly positively correlated with the volatility of market belief since approximately 84% of $\hat{\beta}_i$ are positive while 61% exceed the 5% critical value in one-tailed test, and the cross-sectional average of $\hat{\beta}_i$ is 662.7 with t-statistics equal to 7.09. Furthermore, the explanatory power of the typical individual regression is impressive provided that the average R^2 is more than 6.8%.

Stock liquidity generally declines when market belief becomes more volatile, this is shown in the second column of Panel C: over three quarters of $\hat{\beta}_i$ are positive while 42% exceed the 5% critical value in one-tailed test, and the cross-sectional average of $\hat{\beta}_i$ is as well significantly positive. On average, the volatility of market belief can approximately explain 3%-4% of the variation of stock liquidity.

As plotted in Figure 8, there exists a time trend in stock turnover rate which has been increasing over the 1993-2008 period and is then not stationary. For this reason, the empirical studies of trading volume use some forms of detrending to induce stationarity. There exist several detrending skills, including linear detrending, log-linear detrending, first differencing, etc[10]. In this paper, we use the method of first differencing to detrend stock turnover rate, and the regression results for the detrended stock turnover rate are summarized in Panel A. Trading volume decreases with the volatility of market belief for over 50% of sample stocks, but the correlation between the volatility of market belief and trading volume is rather weak because only less than 2% of $\hat{\beta}_i$ are significantly negative. Moreover, the explanatory power of the volatility of market belief on trading volume is least impressive in that the average R^2 is even less than 0.5%. In contrast to the theoretical prediction in Section 3.2.5, the average of $\hat{\beta}_i$ is positive although it is insignificant. All these evidences show that there should be a large component of noise and/or other influences in stock trading activity.

It is possible that the above results depend on the specification of the time-series models used to predict EPS, different EPS predicting models may lead to completely different conclusions. Therefore, as a robustness check, the regression results for the case in which market belief is constructed using the seasonal random walk with drift model are presented in the third column of Table 6. Obviously, the results in this case are very similar to those obtained previously. Indeed, in this case, the results are even stronger to support the theoretically predicted relations between the volatility of market belief about stock payoffs and the trading volume, price volatility and liquidity of underlying stocks in the sense that more $\hat{\beta}_i$ exhibit theoretically predicted signs and are significant, and specially, the average of $\hat{\beta}_i$ in the '*trading volume*' case is now negative and is significant at the 5% level.

3.3.6.2 Multivariate Tests

Although the empirical results so far support the theoretically predicted relations between the volatility of market belief about stock payoffs and the trading volume, price volatility and liquidity of underlying stocks, these relations remain to be further investigated since other factors than the volatility of market belief, which may influence trading volume, price volatility and liquidity, are ignored.

A. Trading volume

To study the behavior of trading volume is a crucial topic in the finance literature provided that trading volume is one of the fundamental building blocks of any theory of market interactions and is important in modeling asset markets. A prior, many empirical studies focusing on the time-series behavior of trading volume document the positive price/volume and volatility/volume relations. Different theoretical models (like sequential arrival of in-

[10]Refer to Lo and Wang (2001) for more discussions about these methods.

formation models, a mixture of distributions models, asymmetric information models, and differences in opinion models) have been developed to explain the relation between price and trading volume, and these models predict a positive relationship between price and trading volume. Some studies also report bidirectional causality between price and trading volume[11]. The mixture of distribution models explains the positive relation between return volatility and trading volume as they jointly depend on a common factor, i.e., information innovation. The third control variable is market belief Z_t. Although the role of Z_t on trading volume is not specified in the theoretical model, according to the experience learned from the financial markets, it seems reasonable to believe that trading volume will increase when the market expectation about the future earnings is optimistic.

Table 7 reports the cross-sectional statistics of the estimates for the betas of regressors. It is clear that including additional explanatory variables does not have a major effect on the negative correlation between trading volume and the volatility of market belief. Very interestingly, the estimated betas of the volatility of market belief are significant at the 5% level for more stocks and its cross-sectional average turns to be negative although still insignificant after controlling for the effects of stock price, return volatility and market belief on trading volume. The average betas of stock price and return volatility exhibit the same signs as we expect, and specially, trading volume strongly positively covaries with return volatility. Trading volume typically increases with market belief of which influence is however much less significant than other two control variables. Including additional explanatory variables control variables makes the explanatory power of the typical individual regression increase from less than 0.5% to more than 8%, this is a huge improvement.

B. Price Volatility

There exists extensive evidence on the relation between price volatility and trading volume. Karpoff (1987), for example, cities many studies that document a positive relation between price volatility and trading volume in financial markets, and this relation is robust to various time intervals and numerous financial markets. For this reason, stock turnover rate is added as a control variable when we run the regression of price volatility on the volatility of market belief. The second control variable is size which is defined as the market value (in billions of U.S. dollars) of stock measured on the last day of the previous month. *size* is included for the following reason: size is correlated with institutional ownership, Falkenstein (1996) finds that institutional investors display a revealed preference for larger firms. Dennis and Strickland (2004) further find that firm-level volatility is positively related to increased institutional

[11] See, for example, Hiemstra and Jones (1994); Chen, Firth, and Rui (2001); Ratner and Leal (2001)

ownership. Cheung and Ng (1998) find that size is positively correlated with price volatility. Like in the '*trading volume*' case, market belief Z_t is also included as a control variable.

The robustness test results are reported in Table 8, these results confirmed the previously documented findings that price volatility increases with turnover rate, but the correlation is insignificant. The results about the effect of size on price volatility are mixed while the average betas of size are not different from zero with high probabilities. Price volatility is also positively correlated with market belief Z_t, this result somehow contradicts our intuition: if bull market is accompanied with high Z_t and bear market low Z_t, then price volatility should decrease with market belief. One of potential explanations for this result is that optimistic market belief, as shown in the robustness test results for *trading volume*, increases trading volume which will in turn drive price volatility up. Importantly, after controlling for the effects of additional explanatory variables, the positive correlation between price volatility and the volatility of market belief remains for most of sample stocks given that more than 75% of estimated betas of the volatility of market belief are positive while 55% exceed the 5% one-tailed critical value. On average, the factors of the volatility of market belief, size, trading volume and the market state of belief can explain approximately 15% of the variation of price volatility, and a minor half of the explanatory power is from the volatility of market belief.

C. Liquidity

Market microstructure theory suggests two possible influencing factors on stock liquidity, namely, inventory risk and asymmetric information. The inventory explanation for liquidity argues that more trading should lead to tight spread because inventory balances and risk per trade can be maintained at lower levels. To account for the influence of inventory risk on liquidity, we include stock turnover rate, which is a measure of trading activity, as a control variable. Sadka and Scherbina (2009) argue that investors disagree more when the problem of asymmetric information is more serious, we can thus use the analyst forecast dispersion (available in the I/B/E/S) as a proxy variable for information asymmetry[12]. Moreover, as suggested by Chordia, Roll, and Subrahyanman (2000), the lagged, contemporaneous and leading market return and return volatility are also included as additional control variables. The market return is intended to remove the spurious dependence induced by an association between returns and liquidity measure. This could have particular relevance for the effective spread since it is a function of the transaction price. Its changes are functions of individual

[12]The data of other proxy variables for asymmetric information, like number of transactions and PINs, are not available.

returns, known to be significantly correlated with the board market returns. The lags and leads are designed to capture any lagged adjustment in market return. The return volatility is a nuisance variable possibly influencing liquidity. The last control variable is market belief Z_t.

Table 9 reports the cross-sectional statistics for the robustness test results. It is evident that including additional explanatory variables does not have a major effect on the positive correlation between stock liquidity and the volatility of market belief: effective spread is positively correlated with the volatility of market belief for about two thirds of sample stocks and the positive correlation is significant for more than one third of sample stocks. The effects of all control variables but *return volatility* on stock liquidity are however opposite to what expected. For example, larger trading volume widens effective spread. This result is not so surprise as it looks like. As shown in Table 8, trading volume significantly increases price volatility which can in turn widen effective spread. Similar argument can be applied to explain the positive correlation between effective spread and market belief and the market return since trading volume likely increases when the market is optimistic and/or when the financial markets boom. Effective spread unexpectedly decreases with the analyst forecast dispersion, suggesting that either the analyst forecast dispersion is not a good proxy variable for information asymmetry or effective spread is not greatly affected by information asymmetry.

3.3.6.3 Volatility of Market Belief Estimated with GARCH(1,1) Model

All the results about the relations between the volatility of market belief about stocks payoffs and the trading volume, price volatility and liquidity of underlying stocks obtained so far are mainly based on the volatility of market belief estimated using rolling regression method. To verify whether these results depend on the specification of estimating methods for the volatility of market belief, this paper also reports in Table 10 the cross-sectional statistics of the relevant regression results for the case in which the volatility of market belief is estimated using a GARCH(p, q) model with orders $p = q = 1$ (as specified in Section 3.3.4).

The main conclusions to be made from Table 10 can be summarized as follows:

First, the theoretically predicted relations between the volatility of market belief about stock payoffs and the trading volume, price volatility and liquidity of underlying stocks hold again: for most of sample stocks, trading volume and liquidity decrease but price volatility increases with the volatility of market belief. Furthermore, the negative correlation between trading volume and the volatility of market belief holds and is significant for more stocks, and the average betas are now significantly negative in all cases;

Second, the volatility of market belief, estimated using GARCH(1,1) model, can explain much more proportion of the variations of price volatility and stock liquidity. For instance, in the simple regressions without any additional control variables, the average R^2 approximately increases from 7% to 14% in the '*price volatility*' case and from 3.5% to 7% in the '*liquidity*' case, almost doubled.

Third, the magnitude of the correlation between the volatility of market belief and price volatility and liquidity varies dramatically across sample stocks, maybe this can explain why the average betas of the volatility of market belief in both the '*price volatility*' and '*liquidity*' cases are less significant although their values are larger than those obtained when we use the volatility of market belief estimated with rolling regression method as explanatory variable.

Using the volatility of market belief estimated with GRACH model does not change the conclusions.

3.3.6.4 Alternative Liquidity Measure

In order to check the robustness of the positive correlation between the volatility of market belief and stock illiquidity, we shall use another liquidity measure, namely, Amihud's (2002) liquidity measure in the empirical test. This measure on day d for stock j is defined as the ratio of its absolute daily return to the daily trading volume (in billions of U.S. dollars), and it is designed to capture the price impact of the order flow. The average illiquidity of stock j in month m, denoted by $ILLIQ_{jm}$, can be formulated as follows

$$ILLIQ_{jm} = \frac{1}{D_{jm}} \sum_{d=1}^{D_{jm}} |R_{jmd}|/VOL_{jmd} \tag{3.46}$$

where D_{jm} is the number of trading days in month m for stock j, R_{jmt} is the return on stock j on day d of month m and VOL_{jmd} is the respective daily trading volume (in hundred millions of U.S. dollars). Amihud (2002) shows that this liquidity measure is positively related to measures of price impact and fixed transaction costs. To avoid the potential non-stationarity of this measure caused by market capitalization growth over the sample period, $ILLIQ_{jm}$ will be multiplied by a scaling factor which equals the ratio of the market capitalization of stock j in the beginning of month m to its value in the last month of the sample period.

The cross-sectional statistics of time-series means of $ILLIQ$ is reported in the column 5 of Table 5 while the bottom right graph in Figure 8 traces the time-series of $ILLIQ$s. We observe that, for a specific stock, the scaling factor of *hundred millions of U.S. dollars* is so large that its $ILLIQ$ value is much larger than the $ILLIQ$ values of other stocks, this is why the graph of $ILLIQ$ looks uncomfortable.

The cross-sectional statistics of the regression results for $ILLIQ$ are reported in Table 11. Let's first consider the case in which the volatility of market belief is the unique explanatory variable. The correlation between the volatility of market belief and stock liquidity is again negative for a major number of sample stocks and significant for about 30% of sample stocks, and the explanatory power of the typical individual regression varies within a similar range as before. One exception is that the cross-sectional average betas of the volatility of market belief turn to be negative, but, they are insignificant.

After controlling for the effects of additional explanatory variables on $ILLIQ$, the number of positive betas of the volatility of market belief even increases in three out of four robustness regressions, and the percentage of significantly positive betas declines to 18%-26% which are still fairly high. Note that, in this case, the signs of the average betas of all control variables but *return volatility* are reversed. Now, stock liquidity significantly positively covaries with trading volume, this result may be not strange as, according to the definition (3.48), $ILLIQ$ is inversely related to trading volume. As expected, stock liquidity decreases with the analyst forecast dispersion. Indeed, both $ILLIQ$ and the analyst forecast dispersion are supposed to be positively correlated with a common factor – information asymmetry.

The results for $ILLIQ$, although impressive, are less significant than those obtained for the Roll's (1984) bid-ask effective spread, this may be because $ILLIQ$, as a measure of the price impact of order flow, is not a pure price pressure measure and therefore contains other components on which the volatility of market belief has less effect.

3.4 Conclusions

This paper studies the impact of investors' heterogeneous beliefs on the trading volume, price volatility and liquidity of stocks. Following Kurz and Motolese (2008), a simple theoretical model is proposed to demonstrate that stock price is linearly positively correlated with market belief about stock future earnings. Moreover, trading volume and liquidity are shown to decrease with the volatility of market belief while price volatility increases with it. Using the analyst forecast data on quarterly EPS provided by the $I/B/E/S$, we find the evidence to support these theoretical predictions. The empirical results are robust to diverse methods of estimating market belief and its volatility and to alternative liquidity measures.

The stocks selected in this paper are from S&P500 index and are generally issued by large publicly held companies. It will be very nature to extend the sample to include more stocks, particularly those of small and medium-sized companies, to check whether the conclusions made in this paper hold for wider range of stocks. Having more stocks also makes it possible for us to study whether the relations between the volatility of market belief and the trading volume, price volatility and liquidity of stocks vary across industry categories and differen-sized firms.

As shown, market belief, in addition to its volatility, also influence the trading volume, price volatility and effective spread of stocks, but theoretically the relations among these variables are not justified. This issue needs to be explored in more details in the future.

3.5 Appendix

3.5.1 Proof of Proposition II

For simplicity, we ignore in this appendix index i identifying the investor who carries out the optimization. The dynamic programming problem is as follows: given initial values (θ_0, W_0), maximize

$$U_t = E_{\theta,c}\left[\sum_{s=0}^{\infty} -\beta^{t+s-1}e^{-\frac{1}{\tau}C_{t+s}}|\Im_t, g_t\right]$$

subject to the following constraints

$$\begin{aligned} W_{t+1} &= (W_t - C_t)\,R + \theta_t Q_{t+1} \\ Q_{t+1} &= p_{t+1} + (d_{t+1} + \mu) - p_t R \end{aligned}$$

and $\psi_t = (1, d_t, Z_t, g_t)^T$. The stochastic transition functions for d_t, Z_t and g_t are defined as

$$\begin{aligned} d_{t+1} &= \lambda_d d_t + \lambda_g^d g_t + \epsilon_{t+1}^d \\ Z_{t+1} &= \lambda_Z Z_t + \lambda_g^Z g_t + \epsilon_{t+1}^Z \\ g_{t+1} &= \lambda_Z g_t + \epsilon_{t+1}^g \end{aligned}$$

In the following, we define that

$$\Lambda_\psi = \begin{pmatrix} 1 & \mathbf{0}^T \\ & \\ \mathbf{0} & \Lambda \end{pmatrix},\ \epsilon_t = \begin{pmatrix} 1 \\ \\ \hat{\epsilon}_t \end{pmatrix},\ \hat{\epsilon}_t = \begin{pmatrix} \epsilon_t^d \\ \epsilon_t^z \\ \epsilon_t^g \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \hat{\sigma}_d^2 & \hat{\sigma}_{dZ} & 0 \\ \hat{\sigma}_{dZ} & \hat{\sigma}_Z^2 & 0 \\ 0 & 0 & \sigma_g^2 \end{bmatrix}\right)$$

Step One: Simplification We define that, for an unknown symmetric matrix V

$$\Lambda = \begin{pmatrix} \lambda_d & 0 & \lambda_g^d \\ 0 & \lambda_z & \lambda_g^z \\ 0 & 0 & \lambda_z \end{pmatrix},\ V = \begin{pmatrix} v_{00} & v_0^T \\ & \\ v_0 & V_{11} \end{pmatrix}$$

Hence $\psi_{t+1} = \Lambda_\psi \psi_t + \Lambda_\epsilon \epsilon_{t+1}$, Λ_ϵ being a 4×4 matrix with a $I_{3\times 3}$ matrix in the right bottom and zeros otherwise.

Assume that $p_t = P_0 + a_d d_t + a_z Z_t$ and we will verify this formula later when we solve for the equilibrium. Using this price formula, we can compute the excess share return in terms of the state variables

$$\begin{aligned} Q_{t+1} &= a_d d_{t+1} + a_z Z_{t+1} + P_0 + d_{t+1} + \mu - (a_d d_t + a_z Z_t + P_0) R \\ &= a^T \psi_t + b^T \epsilon_{t+1} \end{aligned}$$

where a and b are defined as follows

$$\begin{aligned} a &= \left(P_0 (1-R) + \mu, (a_d+1) \lambda_d - R a_d, a_z \lambda_z - R a_z, (a_d+1) \lambda_g^d + a_z \lambda_g^z\right)^T \\ b &= (0, a_d+1, a_z, 0)^T \end{aligned}$$

Hence, we have that $E_t (Q_{t+1}) = a^T \psi_t$. Also, we use the notation $\hat{b} = (a_d+1, a_z, 0)^T$. Consider the following trial solution for the value function of the dynamic programming problem

$$J (W_t; \psi_t; t) = -\beta^{t-1} exp \left\{ -\alpha W_t - \frac{1}{2} \psi_t^T V \psi_t \right\}$$

Compute the expression

$$\begin{aligned} -\alpha W_{t+1} - \frac{1}{2} \psi_{t+1}^T V \psi_{t+1} &= -\alpha (W_t - C_t) R - \alpha \theta_t \left[a^T \psi_t + b^T \epsilon_{t+1}\right] \\ &\quad -\frac{1}{2} \psi_t^T \Lambda_\psi^T V \Lambda_\psi \psi_t - \psi_t^T \Lambda_\psi^T V \Lambda_\epsilon \epsilon_{t+1} - \frac{1}{2} \epsilon_{t+1}^T \Lambda_\epsilon^T V \Lambda_\epsilon \epsilon_{t+1} \\ &= -A_t - e_t^T \hat{\epsilon}_{t+1} - \frac{1}{2} \hat{\epsilon}_{t+1}^T V_{11} \hat{\epsilon}_{T+1} \end{aligned}$$

where A_t and e_t are defined as

$$\begin{aligned} A_t &= \alpha (W_t - C_t) R + \alpha \theta_t a^T \psi_t + \frac{1}{2} \psi_t^T \Lambda_\psi^T V \Lambda_\psi \psi_t \\ e_t &= \left(\alpha \theta_t \hat{b}^T + \psi_t^T \Lambda_0^T\right)^T \end{aligned}$$

with $\Lambda_0 = (v_0, V_{11} \Lambda)$, which is a (3x4) matrix.

Step Two: The Bellman Equation The Bellman equation for this problem with $\gamma = \frac{1}{\tau}$

can be written in the form

$$\begin{aligned}J_t &= Max_{\{\theta_t,C_t\}}\left[-\beta^{t-1}exp\{-\gamma C_t\} + E\left(J_{t+1}|\Im_t, g_t\right)\right]\\ &= Max_{\{\theta_t,C_t\}}\left[-\beta^{t-1}exp\{-\gamma C_t\} - \beta^t E_t\left(exp\left\{-A_t - e_t^T\hat{\epsilon}_{t+1} - \frac{1}{2}\hat{\epsilon}_{t+1}^T V_{11}\hat{\epsilon}_{t+1}\right\}\right)\right]\end{aligned}$$

It can be shown that

$$E_t\left(exp\left\{-A_t - e_t^T\hat{\epsilon}_{t+1} - \frac{1}{2}\hat{\epsilon}_{t+1}^T V_{11}\hat{\epsilon}_{t+1}\right\}\right) = \left|1+\sum V_{11}\right|^{-\frac{1}{2}} exp\left[\frac{1}{2}e_t^T\left(1+\sum V_{11}\right)^{-1}\sum e_t - A_t\right]$$

Also

$$\begin{aligned}\frac{1}{2}e_t^T\left(1+\sum V_{11}\right)^{-1}\sum e_t &= \frac{1}{2}\left(\alpha\theta_t\hat{b}^T + \psi_t^T\Lambda_0^T\right)\left(1+\sum V_{11}\right)^{-1}\sum\left(\alpha\theta_t\hat{b}^T + \psi_t^T\Lambda_0^T\right)^T\\ &= \frac{1}{2}\alpha^2\theta_t^2\hat{b}^T\Omega\hat{b} + \alpha\theta_t\hat{b}^T\Omega\Lambda_0\psi_t + \frac{1}{2}\psi_t^T\Lambda_0^T\Omega\Lambda_0\psi_t\end{aligned}$$

where $\Omega = \left(1+\sum V_{11}\right)^{-1}\sum$.
Hence, we have an expression for the exponent term

$$\begin{aligned}\frac{1}{2}e_t^T\left(1+\sum V_{11}\right)^{-1}\sum e_t - A_t &= -\alpha\left(W_t - C_t\right)R - \alpha\theta_t\left[a^T - \hat{b}^T\Omega\Lambda_0\right]\psi_t\\ &\quad + \frac{1}{2}\alpha^2\theta_t^2\hat{b}^T\Omega\hat{b} - \frac{1}{2}\psi_t^T\left[\Lambda_\psi^T V\Lambda_\psi - \Lambda_0^T\Omega\Lambda_0\right]\psi_t\end{aligned}$$

The first order condition with respect to θ leads to

$$-\alpha\left[a^T - \hat{b}^T\Omega\Lambda_0\right]\psi_t + \alpha^2\theta_t\hat{b}^T\Omega\hat{b} = 0$$

The demand of each investor thus equals (since $E_t\left(Q_{t+1}\right) = a^T\psi_t$)

$$\theta_t = \frac{1}{\alpha\hat{b}^T\Omega\hat{b}}\left[\left(a^T - \hat{b}^T\Omega\Lambda_0\right)\psi_t\right] = \frac{1}{\alpha\hat{b}^T\Omega\hat{b}}\left[E_t\left(Q_{t+1}\right) + u^T\psi_t\right]$$

with $u^T = -\hat{b}^T\Omega\Lambda_0$.
Note that the vector u is the same for the all investors since the assumption made in the text is that all investors are identically the same except for their belief states g_t and the last equation shows that the vector u depends only upon parameters of the stochastic structure.

Step Three: The Adjusted Variance and Constants Define that

$$\hat{\sigma}_Q^2 = \hat{b}^T \Omega \hat{b}$$

which is the variance of the excess return function where the covariance matrix used is not $\sum$ but rather Ω. We now have

$$\alpha^2 \theta_t^2 \hat{b}^T \Omega \hat{b} = \frac{1}{\hat{b}^T \Omega \hat{b}} \left\{ \psi_t^T \left[a^T - \hat{b}^T \Omega \Lambda_0 \right]^T \left[a^T - \hat{b}^T \Omega \Lambda_0 \right] \psi_t \right\}$$

Hence the optimized value of the exponent is simply

$$\frac{1}{2} e_t^T \left(1 + \sum V_{11}\right)^{-1} \sum e_t - A_t = -\alpha \left(W_t - C_t\right) R - \tfrac{1}{2} \psi_t^T M \psi_t$$

where

$$M = \frac{1}{\hat{b}^T \Omega \hat{b}} \left[a^T - \hat{b}^T \Omega \Lambda_0 \right]^T \left[a^T - \hat{b}^T \Omega \Lambda_0 \right] + \left[\Lambda_\psi^T V \Lambda_\psi - \Lambda_0^T \Omega \Lambda_0 \right]$$

Now take the derivative with respect to C_t and equate to zero to obtain

$$\gamma exp \left\{ -\gamma C_t \right\} = \alpha R \beta \left| 1 + \sum V_{11} \right|^{-\frac{1}{2}} exp \left\{ -\alpha \left(W_t - C_t\right) R - \frac{1}{2} \psi_t^T M \psi_t \right\}$$

Let $G = \left| 1 + \sum V_{11} \right|^{-\frac{1}{2}}$. Hence, the solution for C_t must satisfy

$$\gamma C_t = -ln \left[\frac{\alpha R \beta G}{\gamma} \right] + \alpha \left(W_t - C_t\right) R + \frac{1}{2} \psi_t^T M \psi_t$$

We finally have

$$C_t = -\frac{1}{\gamma + \alpha R} ln \left[\frac{\alpha R \beta G}{\gamma} \right] + \frac{\alpha R}{\gamma + \alpha R} W_t + \frac{1}{2\left(\gamma + \alpha R\right)} \psi_t^T M \psi_t$$

Substituting the optimal consumption-investment policy back into the Bellman equation, we obtain

$$\alpha = \frac{r\gamma}{R}$$

$$exp \left\{ -\frac{1}{2} \psi_t^T \left[\frac{M}{R} - V + 2 \left(\gamma \overline{C} + ln \left(\frac{r}{R} \right) \right) \mathbf{i}_{11} \right] \psi_t \right\} = 1$$

where $\mathbf{i}_{11}$ is a 4×4 matrix with the element $(1,1)$ being one and all other elements being zero and $\overline{C} = -\frac{1}{\gamma R} ln\left(r\beta G\right)$. This leads to the following equation for V[13]

$$\frac{M}{R} - V + 2\left(\gamma\overline{C} + ln\left(\frac{r}{R}\right)\right)\mathbf{i}_{11} = 0$$

Step Four: The Equilibrium Pricing Average θ_t over all investors, given that the total supply of stock is one, we obtain

$$\frac{r\hat{\sigma}_Q^2}{RN\tau} = \left[\overline{E}_t\left(p_{t+1} + d_{t+1} + \mu\right) - Rp_t + \left(u_0 + u_1 d_t + (u_2 + u_3)Z_t\right)\right]$$

Use the relationships (3.19) and (3.20) to deduce a linear difference equation for p_t

$$\overline{E}_t\left(p_{t+1}\right) = Rp_t - \left(\lambda_d + u_1\right)d_t - \left(\lambda_d^g + u_2 + u_3\right)Z_t + \frac{r\hat{\sigma}_Q^2}{RN\tau} - \left(\mu + u_0\right)$$

This is a typical linear difference problem. The parameters and the market average processes of d_t and Z_t satisfy the conditions, as specified in Blanchard and Kahn (1980), under which there does exist an unique equilibrium solution for stock price which, like Eq. (3) in Blanchard and Kahn (1980), takes a form as follows

$$p_t = \sum_{i=0}^{\infty} R^{-i-1}\left\{\gamma_1^T \overline{E}\left(X_{t+i} | \Im_t\right)\right\} \tag{3.47}$$

where

$$\begin{aligned} \gamma_1 &= \left[(\mu + u_0) - \frac{r\hat{\sigma}_Q^2}{RN\tau},\ \lambda_d + u_1,\ \lambda_d^g + u_2 + u_3\right]^T \\ X_t &= \left[1,\ d_t,\ Z_t\right]^T \end{aligned}$$

The price function in Eq. (3.20) can be obtained by simplifying the price formula (3.47).

[13]The equation (ii) determining V in the appendix A of Kurz and Motolese (2008) seems incorrect.

3.5.2 Some Moments of Absolute Normal Random Variables

Let Y be a normally distributed random variable with mean μ and σ^2, that is, $Y \sim N(\mu, \sigma^2)$. The expectation of the absolute value of Y is:

$$\begin{aligned}
E(|Y|) &= \int_{-\infty}^{+\infty} \frac{|y|}{\sqrt{2\pi}\sigma} exp\left[-\frac{(y-\mu)^2}{2\sigma^2}\right] dy \\
&= \int_{-\mu}^{+\infty} \frac{x+\mu}{\sqrt{2\pi}\sigma} exp\left(-\frac{x^2}{2\sigma^2}\right) dx - \int_{-\infty}^{-\mu} \frac{x+\mu}{\sqrt{2\pi}\sigma} exp\left(-\frac{x^2}{2\sigma^2}\right) dx \\
&= 2\int_{-\mu}^{+\infty} \frac{x}{\sqrt{2\pi}\sigma} exp\left(-\frac{x^2}{2\sigma^2}\right) dx + \mu\left[1 - 2\int_{-\infty}^{-\mu} \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{x^2}{2\sigma^2}\right) dx\right]
\end{aligned}$$

It is easy to show that

$$\int_{-\mu}^{+\infty} \frac{x}{\sqrt{2\pi}\sigma} exp\left(-\frac{x^2}{2\sigma^2}\right) dx = \sigma\sqrt{\frac{1}{2\pi}} exp\left(-\frac{\mu^2}{2\sigma^2}\right)$$

Thus, we have that the expectation of the absolute value of Y is given by

$$E(|Y|) = \sigma\sqrt{\frac{2}{\pi}} exp\left(-\frac{\mu^2}{2\sigma^2}\right) + \mu\left[1 - 2\Phi\left(-\frac{\mu}{\sigma}\right)\right]$$

The variance of $|Y|$ is easy to calculate with $E(|Y|)$

$$\begin{aligned}
var(|Y|) &= E\left(|Y|^2\right) - E\left(|Y|\right)^2 = var\left(Y^2\right) + E\left(Y\right)^2 - E\left(|Y|\right)^2 \\
&= \mu^2 + \sigma^2 - \left[\sigma\sqrt{\frac{2}{\pi}} exp\left(-\frac{\mu^2}{2\sigma^2}\right) + \mu\left(1 - 2\Phi\left(-\frac{\mu}{\sigma}\right)\right)\right]^2
\end{aligned}$$

These results can be used to calculate the expectations of V_t^i and $|\Lambda_t|$ defined in Section 3.2.

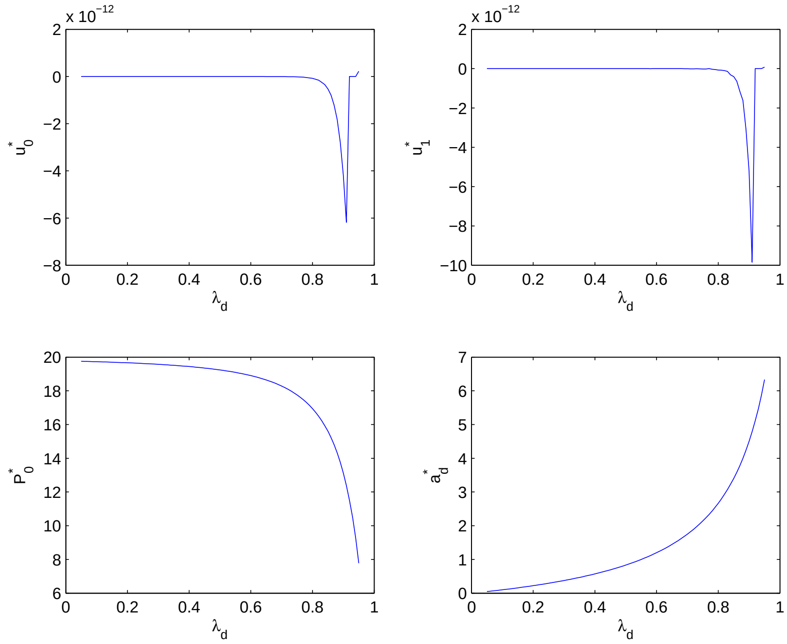

Figure 1A: This figure plots the numerically computed values for the coefficients u_0^*, u_1^*, P_0^* and a_d^* against λ_d with r=0.1, μ=2, β=0.9, τ=2.0, σ_d^2=1.0, N=2.

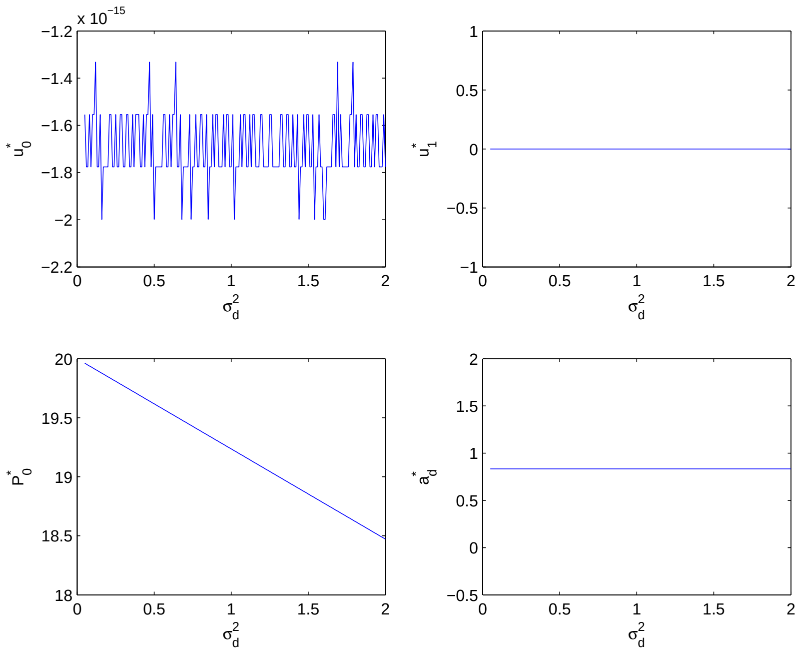

Figure 1B: This figure plots the numerically computed values for the coefficients u_0^*, u_1^*, P_0^* and a_d^* against σ_d^2 with r=0.1, μ=2, β=0.9, τ=2.0, λ_d=0.5, N=2.

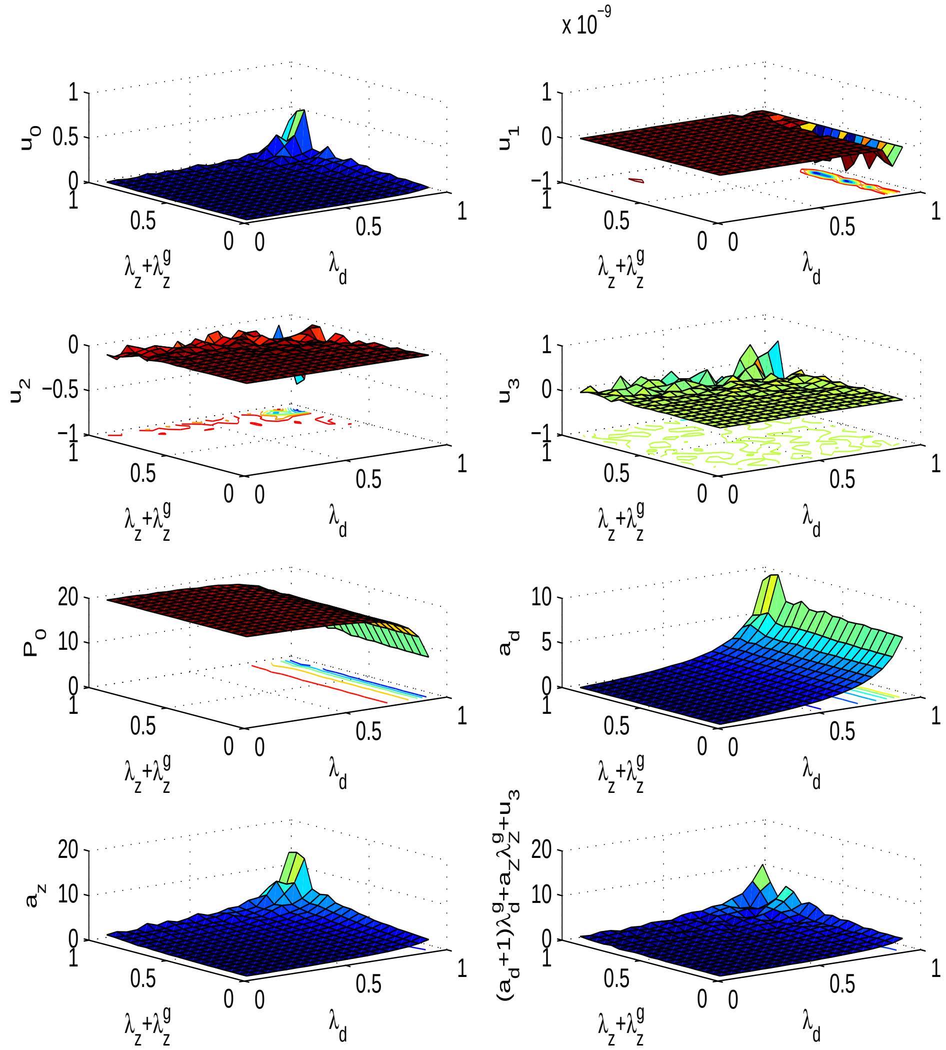

Figure 2A: This figure plots the numerically computed values for the coefficients u_0, u_1, u_2, u_3, P_0, a_d, a_Z and $(a_d+1)\lambda_d^g + a_Z\lambda_Z^g + u_3$ against both λ_d and $\lambda_Z + \lambda_Z^g$ with r=0.1, μ=2, β=0.9, τ=2.0, λ_d^g=0.3, σ_d^2=1.0, $\hat{\sigma}_Z^2$=0.6, σ_g^2=1.0, $\hat{\sigma}_{dZ}$=$\hat{\sigma}_{dg}$=0, N=2.

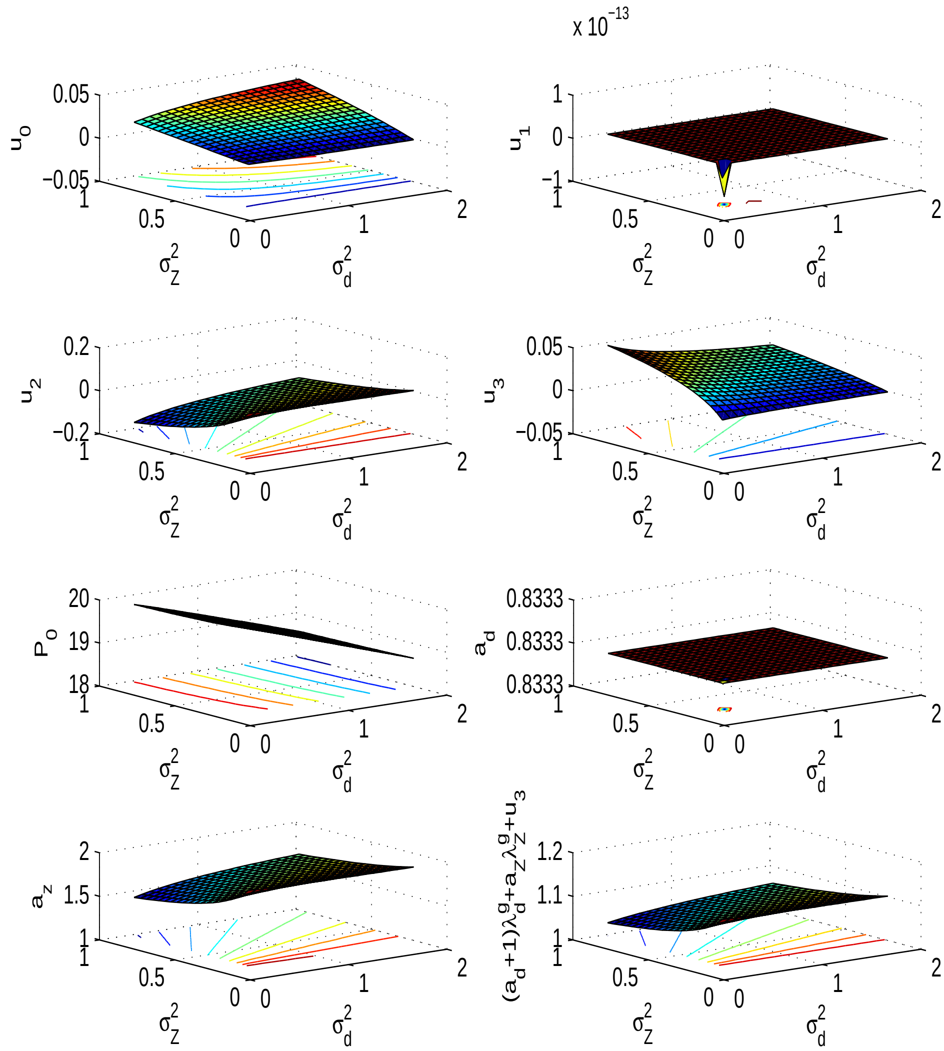

Figure 2B: This figure plots the numerically computed values for the coefficients u_0, u_1, u_2, u_3, P_0, a_d, a_Z and $(a_d+1)\lambda_d^g + a_Z\lambda_Z^g + u_3$ against both σ_d^2 and σ_Z^2 with r=0.1, μ=2, β=0.9, τ=2.0, λ_d=0.5, λ_d^g=0.3, λ_Z=0.5, λ_Z^g=0.3, σ_g^2=1.0, $\hat{\sigma}_{dZ}$=$\hat{\sigma}_{dg}$=0, N=2.

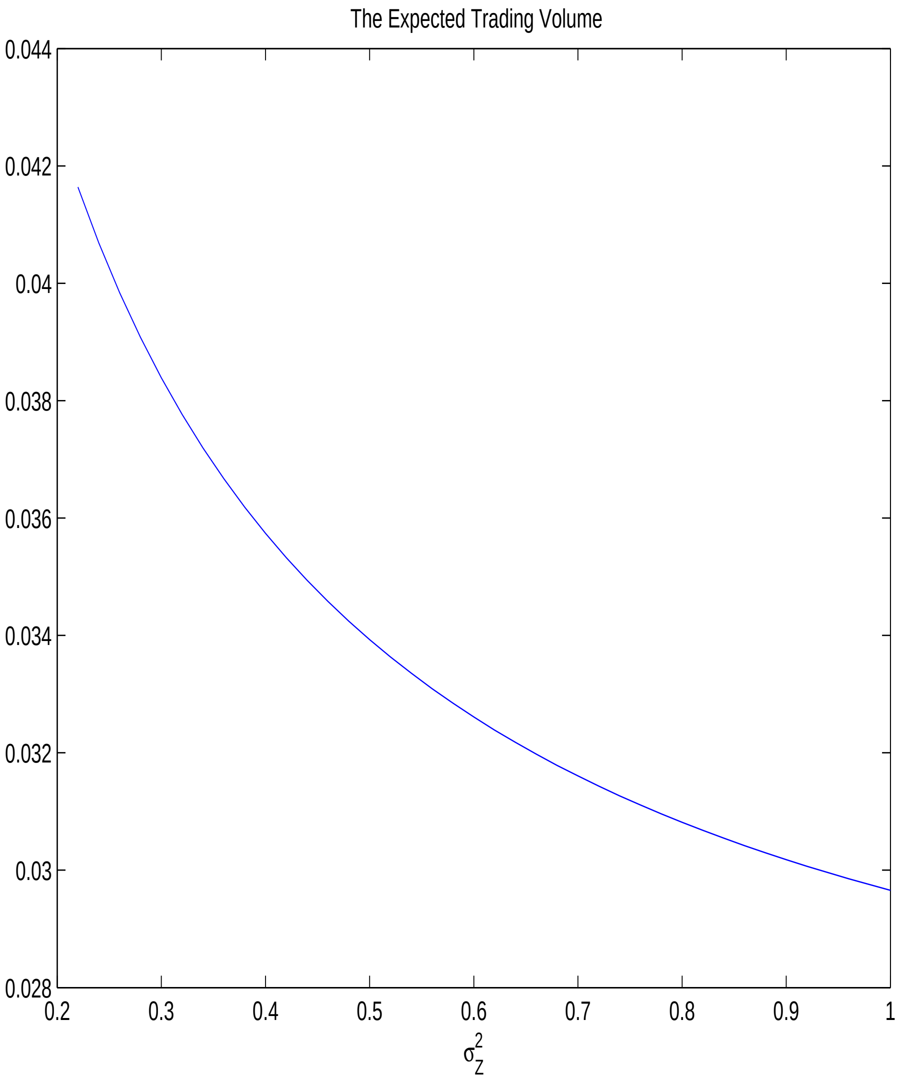

Figure 3: This figure plots the numerically computed expected trading volume $\overline{V}$ against σ_Z^2 with r=0.1, μ=2, β=0.9, τ=2.0, λ_d=0.5, λ_d^g=0.3, λ_Z=0.5, λ_Z^g=0.3, σ_d^2=1.0, σ_g^2=1.0, $\hat{\sigma}_{dZ}$=$\hat{\sigma}_{dg}$=0, N=2.

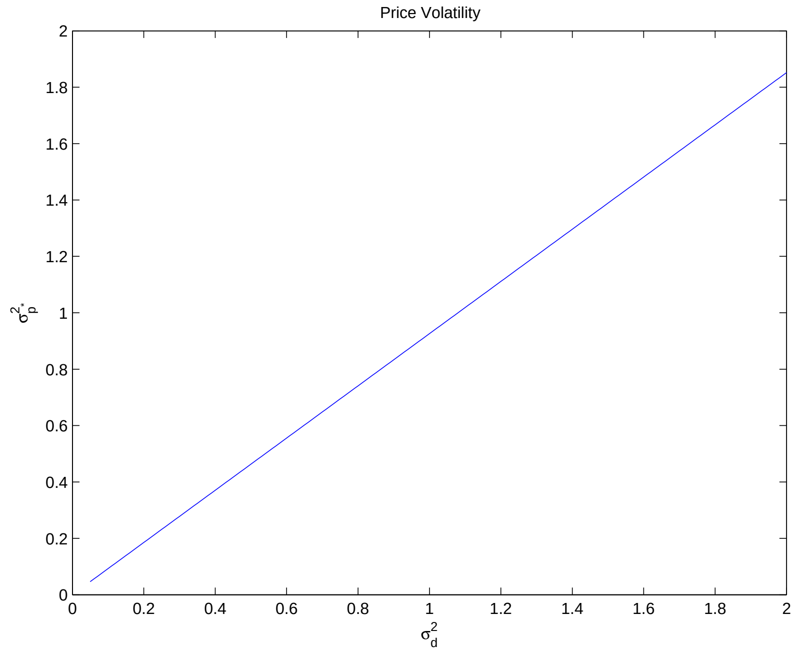

Figure 4A: This figure plots the numerically computed price volatility $\sigma^2_{p^*}$ against σ^2_d with r=0.1, μ=2, β=0.9, τ=2.0, λ_d=0.5, N=2.

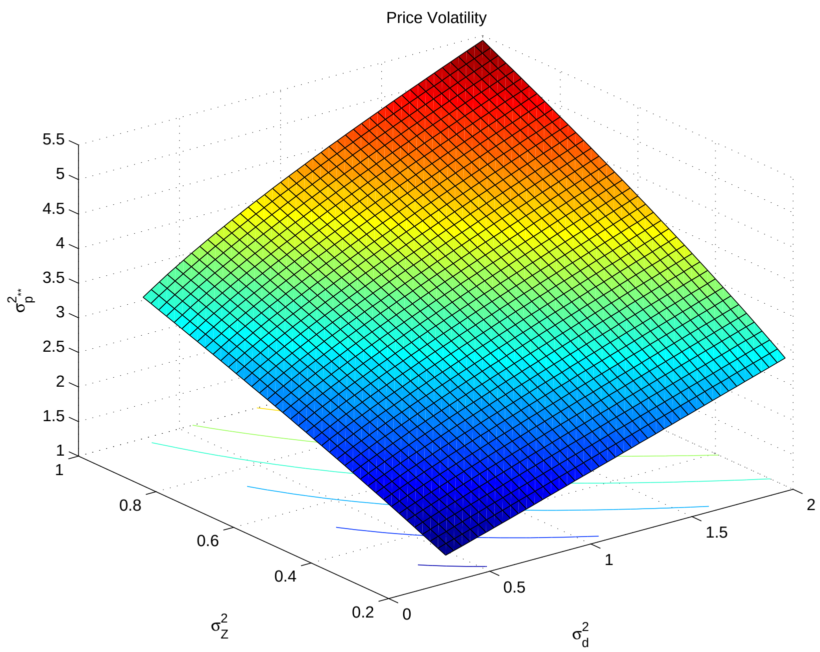

Figure 4B: This figure plots the numerically computed price volatility $\sigma^2_{p^{**}}$ against both σ^2_d and σ^2_Z with r=0.1, μ=2, β=0.9, τ=2.0, λ_d=0.5, λ^g_d=0.3, λ_Z=0.5, λ^g_Z=0.3, σ^2_g=1.0, $\hat{\sigma}_{dZ}$=$\hat{\sigma}_{dg}$=0, N=2.

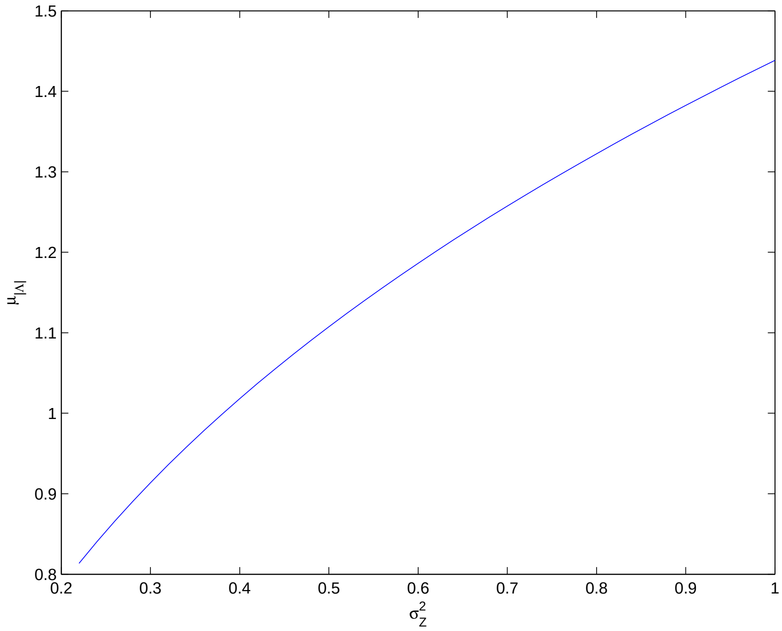

Figure 5: This figure plots the numerically computed unconditional expected liquidity $|\Lambda_t|$ against σ_Z^2 with r=0.1, μ=2, β=0.9, τ=2.0, λ_d=0.5, λ_d^g=0.3, λ_Z=0.5, λ_Z^g=0.3, σ_d^2=1.0, σ_g^2=1.0, $\hat{\sigma}_{dZ}$=$\hat{\sigma}_{dg}$=0, N=2.

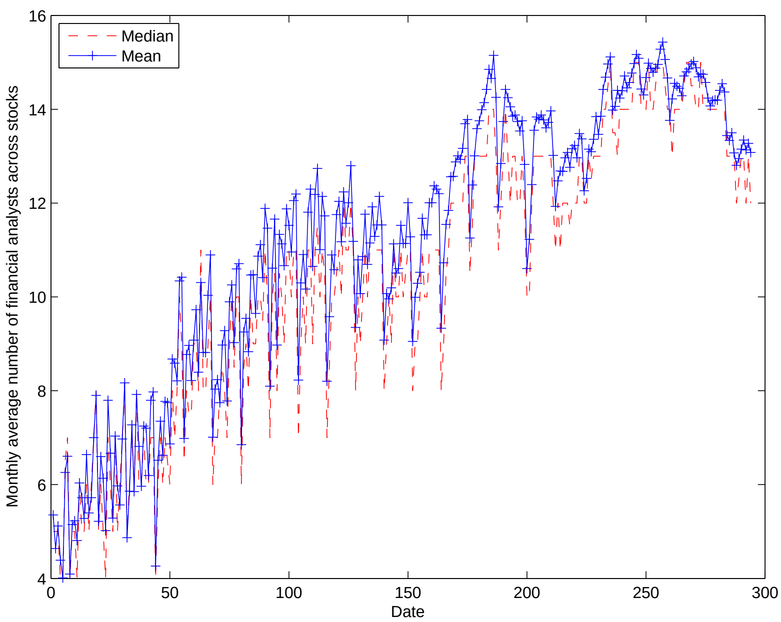

Figure 6: This figure plots, for 114 S&P500 index stocks, the monthly cross-sectional average number (mean or median) of financial analysts over the period from 1984 to 2008.

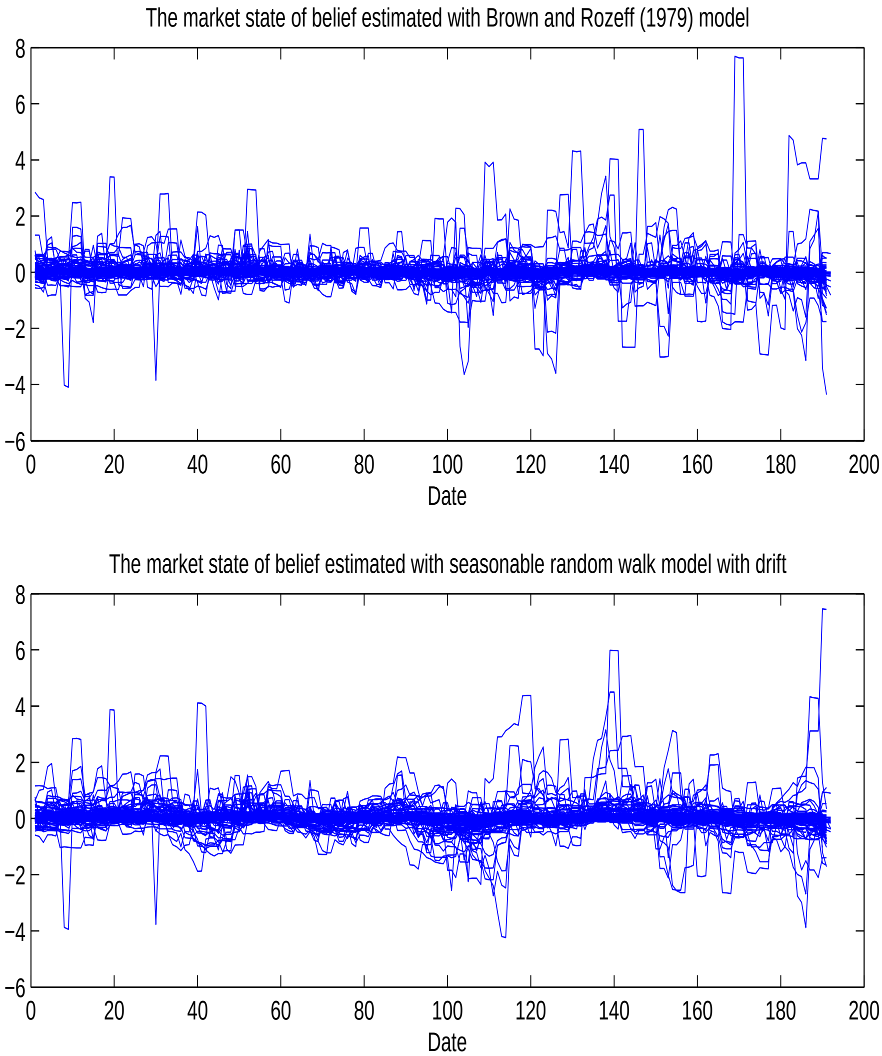

Figure 7: This figure plots, for 114 S&P500 stocks, the time-series of market beliefs estimated with both the Brown and Rozeff (1979) model and the seasonal random walk with drift model over the period from 1993 to 2008.

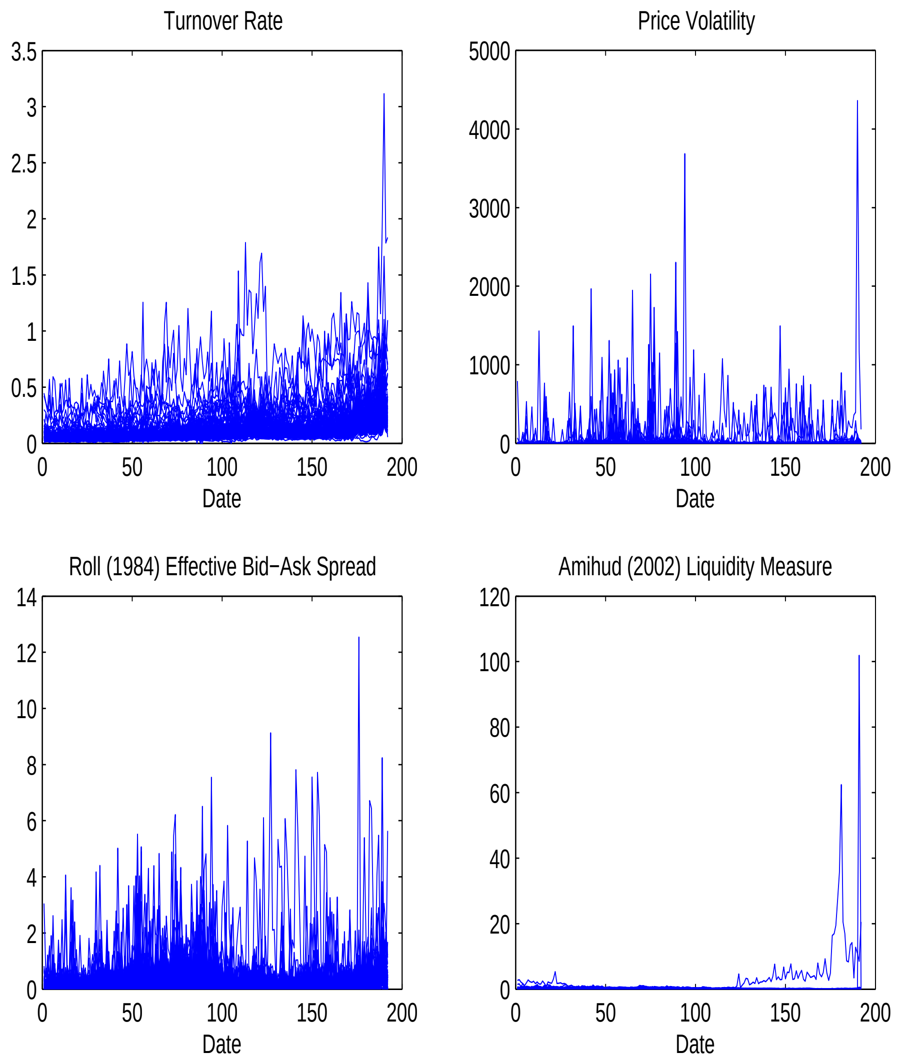

Figure 8: This figure plots, for 114 S&P500 stocks, the time series of dependent variables: turnover rate, price volatility, Roll's (1984) effective bid-ask spread, Amihud's (2002) liquidity measure over the period from 1993 to 2008.

Table 1
G.I.C.S. Industry Breakdown of the Sample.

GICS Two-Digit Industry Code	Sector	Number of Stocks
10	Energy	10
15	Materials	14
20	Industrials	24
25	Consumer Discretionary	19
30	Consumer Staples	14
35	Health Care	11
40	Financials	8
45	Information Technology	12
50	Telecommunication Services	0
55	Utilities	2
		—
	Total	114

Table 2

Cross-sectional statistics of time series means of the estimated coefficients for the Brown and Rozeff (1979) (BR) model and the seasonal random walk with drift (SRWD) model.

This table reports cross-sectional statistics of time series means of the estimated coefficients for both the Brown and Rozeff (1979) model and the seasonal random walk with drift model. The seasonal random walk with drift model can be written as:

$$E(Q_t) = \delta + Q_{t-4} \tag{3.48}$$

where $E(Q_t)$ is the earnings forecast for quarter t, δ is a (typically) positive trend, and Q_{t-4} is the actual earnings for quarter t-4. The Brown and Rozeff model takes a form as follows:

$$E(Q_t) = \delta + Q_{t-4} + \phi(Q_{t-1} - Q_{t-5}) + \theta\epsilon_{t-4} \tag{3.49}$$

where Q_{t-k} is the actual earnings for quarter $t-k$ and ϵ_{t-4} is the white noise earnings shock experienced at quarter $t-4$. The sample consists of 114 S&P500 index stocks.

	SRWD Model	BR Model		
	δ	δ	ϕ	θ
# of negative medians	6	7	0	101
Mean	0.027	0.017	0.560	-0.224
Median	0.024	0.015	0.583	-0.258
Standard Deviation	0.024	0.018	0.191	0.153

Table 3

Cross-sectional statistics of time series means, skewness, and kurtosis of the market state of belief.

This table reports cross-sectional statistics of time series means, skewness and kurtosis of the market state of belief Z_t. The market state of belief is estimated by subtracting the quarterly EPS predicted with both the Brown and Rozeff (1979) model and the seasonal random walk with drift model from the mean of analysts' earnings forecasts, and the estimated beliefs are respectively denoted by Z_{br} and Z_{srwd}. The sample consists of 114 S&P500 index stocks.

	Z_{br}			Z_{srwd}		
	Mean	*Skewness*	*Kurtosis*	*Mean*	*Skewness*	*Kurtosis*
Mean	0.021	1.581	14.996	0.035	0.984	12.612
Median	0.011	1.463	10.987	0.024	0.791	8.864
Standard Deviation	0.051	1.836	11.148	0.048	1.803	10.210

Table 4
Cross-sectional statistics of time series means of the volatility of market belief.

This table reports cross-sectional statistics of time series means of the volatility of market belief estimated using either rolling regression method or GARCH(1,1) model as specified in Section 3.4. The sample consists of 114 S&P500 index stocks.

	$Volatility_{z_{br}}$		$Volatility_{z_{srwd}}$	
	RR Method	*GARCH(1,1)*	*RR Method*	*GARCH(1,1)*
Mean	0.033	0.061	0.020	0.040
Median	0.006	0.012	0.005	0.010
Standard Deviation	0.089	0.161	0.048	0.093

Table 5
Cross-sectional statistics of time series means of dependent variables.

This table reports cross-sectional statistics of time series means of four dependent variables: trading volume (Turnover Rate), price volatility (Price Volatility), Roll's (1984) effective bid ask spread (Effective Spread), and Amihud's (2002) liquidity measure (Price Impact). The sample consists of 114 S&P500 index stocks.

	Turnover Rate	Price Volatility	Effective Spread	Price Impact
Mean	0.124	9.477	0.232	0.102
Median	0.100	6.019	0.207	0.038
Standard Deviation	0.084	18.208	0.145	0.307

Table 6
Explaining the time-series variation of trading volume, price volatility and liquidity of stocks with the volatility of market belief estimated using rolling regression method.

This table reports cross-sectional statistics of the following OLS regression results:

$$Y_{i,t} = \alpha_i + \beta_i vZ^j_{i,t} + \epsilon_{i,t} \quad (i = 1, ..., 114) \tag{3.50}$$

where $j =$ BR *or* SRWD. The dependent variable Y_t is either trading volume or price volatility or effective spread, vZ_t is the volatility of market belief estimated using *rolling regression method*. Trading volume refers to the detrended turnover rate, price volatility is the variance of stock price, effective spread is the square root of negative first order autocovariance of stock price and will be set to zero if the autocovariance is positive. Cross-sectional averages of estimated betas are reported with t-statistics in parentheses. '%positive' ('%negative') means the percentage of positive (negative) betas, while '%+significance' ('%-significance') gives the percentage with t-statistics greater than +1.645 (smaller than -1.645)(the 5% critical value in one-tailed test). The sample consists of 114 S&P500 index stocks.

	BR Model	SRWD Model
Panel A: Trading Volume		
vZ_t	2.777	-5.928
	(0.76)	(-1.66)
% *negative*	58.772	68.421
$\% -significant$	1.754	0.877
$R^2(\%)$ *mean*	0.359	0.336
Median	0.129	0.119
Panel B: Price Volatility		
$vZ_t(/100)$	6.627	7.624
	(7.09)	(6.59)
% *positive*	84.211	85.965
$\% +significant$	60.526	64.912
$R^2(\%)$ *mean*	6.820	7.083
Median	2.989	3.021
Panel C: Liquidity		
vZ_t	3.194	3.642
	(5.53)	(5.49)
% *positive*	75.439	77.193
$\% +significant$	42.105	48.246
$R^2(\%)$ *mean*	3.591	3.677
Median	1.206	1.516

Table 7
Robustness test: trading volume

This table reports cross-sectional statistics of the following OLS regression results:

$$Y_{i,t} = \alpha_i + \beta_{i,1} vZ^j_{i,t} + \beta_{i,2} Z^j_{i,t} + \beta_{i,3} Price_{i,t} + \beta_{i,4} vRet_{i,t} + \epsilon_{i,t} \quad (i = 1, ..., 114) \tag{3.51}$$

where j = BR *or* SRWD. The dependent variable Y_t is the detrended turnover rate, vZ_t is the volatility of market belief estimated using *rolling regression method*, Z_t is the market state of belief, $Price_t$ is the monthly average stock price and $vRet_t$ is the volatility of stock return. Cross-sectional averages of estimated betas are reported with t-statistics in parentheses. '%negative' means the percentage of negative betas, while '%-significance' gives the percentage with t-statistics smaller than -1.645 (the 5% critical value in one-tailed test). The sample consists of 114 S&P500 index stocks.

	BR Model	SRWD Model
vZ_t	-0.682	-8.411
	(-0.19)	(-1.40)
$\%\ negative$	60.526	70.175
$\% -significant$	3.509	5.263
Z_t	0.469	0.736
	(1.27)	(2.28)
$Price_t$	0.017	0.016
	(7.71)	(6.82)
$vRet_t(/100)$	23.571	22.993
	(12.73)	(11.78)
$R^2(\%)\ mean$	7.988	7.902
$Median$	6.951	7.362

Table 8
Robustness test: price volatility

This table reports cross-sectional statistics of the following OLS regression results:

$$Y_{i,t} = \alpha_i + \beta_{i,1} vZ^j_{i,t} + \beta_{i,2} Z^j_{i,t} + \beta_{i,3} Size_{i,t} + \beta_{i,4} Volume_{i,t} + \epsilon_{i,t} \quad (i = 1, ..., 114) \tag{3.52}$$

where j = BR *or* SRWD. The dependent variable Y_t is price volatility, vZ_t is the volatility of market belief estimated using *rolling regression method*, Z_t is the market state of belief, $Size_t$ is the market capitalization of stock in the beginning of each month, and $Volume_t$ is the turnover rate. Cross-sectional averages of estimated betas are reported with t-statistics in parentheses. '%positive' means the percentage of positive betas, while '%+significance' gives the percentage with t-statistics greater than +1.645 (the 5% critical value in one-tailed test). The sample consists of 114 S&P500 index stocks.

	BR Model	SRWD Model
$vZ_t(/100)$	6.244	6.610
	(6.85)	(6.97)
$\%\ positive$	75.439	79.825
$\%\ +significant$	54.386	55.263
Z_t	32.254	30.433
	(3.44)	(3.44)
$Size_t$	0.014	-0.055
	(0.04)	(-0.16)
$Volume_t$	1.497	1.487
	(1.31)	(1.32)
$R^2(\%)\ mean$	16.180	15.938
$Median$	14.174	14.034

Table 9
Robustness test: liquidity

This table reports cross-sectional statistics of the following OLS regression results:

$$\begin{aligned} Y_{i,t} &= \alpha_i + \beta_{i,1} vZ^j_{i,t} + \beta_{i,2} Z^j_{i,t} + \beta_{i,3} MktRet_{i,t-1} + \beta_{i,4} MktRet_{i,t} + \beta_{i,5} MktRet_{i,t+1} \\ &+ \beta_{i,6} vRet_{i,t} + \beta_{i,7} Volume_{i,t} + \beta_{i,8} Dispersion_{i,t} + \epsilon_{i,t} \quad (i = 1, ..., 114) \end{aligned} \tag{3.53}$$

where j = BR *or* SRWD. The dependent variable Y_t is effective spread, vZ_t is the volatility of market belief estimated using *rolling regression method*, Z_t is the market state of belief, $MktRet_{t-1}$, $MktRet_t$ and $MktRet_{t+1}$ are respectively the lag, concurrent and lead values of the value-weighted market return, $vRet_t$ is the volatility of stock return, $Volume_t$ is the turnover rate, and $Dispersion_t$ is the analyst forecast dispersion. Cross-sectional averages of estimated betas are reported with t-statistics in parentheses. '%positive' means the percentage of positive betas, while '%+significance' gives the percentage with t-statistics greater than +1.645 (the 5% critical value in one-tailed test). The sample consists of 114 S&P500 index stocks.

	BR Model	SRWD Model
vZ_t	2.496	2.912
	(4.87)	(5.21)
% positive	66.667	68.421
% +significant	34.211	41.228
Z_t	0.202	0.250
	(4.61)	(5.47)
$MktRet_{t-1}$	0.154	0.150
	(2.29)	(2.22)
$MktRet_t$	0.275	0.272
	(4.42)	(4.38)
$MktRet_{t+1}$	0.050	0.046
	(0.85)	(0.79)
$vRet_t(/100)$	1.528	1.545
	(11.78)	(12.06)
$Volume_t$	0.004	0.004
	(1.22)	(1.20)
$Dispersion_t$	-0.225	-0.223
	(-1.10)	(-0.96)
$R^2(\%)$ *mean*	17.448	17.673
Median	15.351	16.191

Table 10

Explaining the time series variation of trading volume, price volatility and liquidity of stocks with the volatility of market belief estimated using GARCH(1,1) model

This table reports cross-sectional statistics of the following OLS regression results:

$$Y_{i,t} = \alpha_i + \beta_{i,1} vZ^j_{i,t} + \beta_i Vector_{i,t} + \epsilon_{i,t} \quad (i = 1, ..., 114) \tag{3.54}$$

where $j =$ BR *or* SRWD. The dependent variable Y_t is either trading volume or price volatility or effective spread, vZ_t is the volatility of market belief estimated using GARCH(1,1) model. $Vector_t$ is a vector containing different regressors for different dependent variables, and β_i is a coefficient vector. 'Basic' means the case in which the volatility of market belief is the unique explanatory variable while 'Robust' means the robustness test. In the 'Basic' case, β_i is set equal to zero. Cross-sectional averages of estimated betas are reported with t-statistics in parentheses. '%positive' ('%negative') means the percentage of positive (negative) betas, while '%+significance' ('%-significance') gives the percentage with t-statistics greater than +1.645 (smaller than -1.645)(the 5% critical value in one-tailed test). For the parsimonious reason, only the statistics for the betas of vZ_t are reported in this table. The sample consists of 112 S&P500 index stocks.

	BR Model		SRWD Model	
	Basic	Robust	Basic	Robust
Panel A: Trading Volume				
vZ_t	-20.594	-22.782	-36.773	-39.827
	(-3.29)	(-4.00)	(-3.51)	(-3.80)
% negative	74.107	72.321	77.679	76.786
$\% -significant$	3.571	6.250	4.464	8.036
$R^2(\%)$ *Mean*	0.297	8.118	0.319	8.077
Median	0.125	7.180	0.099	7.340
Panel B: Price Volatility				
$vZ_t(/100)$	29.550	29.230	31.948	31.154
	(2.38)	(2.35)	(2.05)	(2.00)
% positive	78.571	75.000	79.464	73.214
$\% +significant$	60.714	58.036	61.607	53.571
$R^2(\%)$ *Mean*	14.262	22.543	13.625	21.482
Median	4.203	15.780	3.758	14.059
Panel C: Liquidity				
vZ_t	14.513	13.793	17.083	16.207
	(2.32)	(2.16)	(2.10)	(1.96)
% positive	76.786	69.643	75.000	66.643
$\% +significant$	48.214	42.857	53.571	40.179
$R^2(\%)$ *Mean*	7.284	20.319	7.252	20.329
Median	1.854	16.034	2.121	16.759

Table 11
Alternative Liquidity Measure

This table reports cross-sectional statistics of the following OLS regression results:

$$Y_{i,t} = \alpha_i + \beta_{i,1} vZ^j_{i,t} + \beta_i Vector_{i,t} + \epsilon_{i,t} \quad (i = 1, ..., 114) \tag{3.55}$$

where j = BR *or* SRWD. The dependent variable Y_t is Amihud's (2002) illiquidity measure, and vZ_t is the volatility of market belief estimated using either rolling regression method or GARCH(1,1) model. $Vector_t$ is a vector containing a set of explanatory variables, and β_i is a beta vector. 'Basic' means the case in which the volatility of market belief is the unique explanatory variable while 'Robust' means the robustness test. β_i is set equal to zero for the 'Basic' case. Cross-sectional averages of estimated betas are reported with t-statistics in parentheses. '%positive' means the percentage of positive betas, while '%+significance' gives the percentage with t-statistics greater than +1.645 (the 5% critical value in one-tailed test). The sample consists of 114 S&P500 index stocks.

	Rolling Regression Method				GARCH(1,1) Model			
	BR Model		SRWD Model		BR Model		SRWD Model	
	Basic	Robust	Basic	Robust	Basic	Robust	Basic	Robust
vZ_t	-0.759	0.484	-0.193	0.353	-0.199	0.118	-0.262	-0.047
	(-1.11)	(0.98)	(-1.01)	(0.99)	(-0.84)	(0.33)	(-0.48)	(-0.11)
% positive	57.54	68.42	60.53	64.91	57.14	62.50	62.50	58.04
% +significant	31.58	17.54	30.70	21.93	26.78	22.32	30.36	25.89
Z_t		-0.086		-0.018		-0.117		-0.033
		(-1.00)		(-0.76)		(-1.01)		(-0.91)
$MktRet_{t-1}$		-0.250		-0.249		-0.253		-0.250
		(-1.03)		(-1.04)		(-1.03)		(-1.04)
$MktRet_t$		-0.405		-0.409		-0.418		-0.421
		(-1.11)		(-1.11)		(-1.11)		(-1.11)
$MktRet_{t+1}$		-0.099		-0.089		-0.104		-0.091
		(-0.83)		(-0.79)		(-0.85)		(-0.80)
$vRet_t(/100)$		0.898		0.886		0.908		0.887
		(1.91)		(1.91)		(1.89)		(1.89)
$Volume_t$		-0.007		-0.007		-0.007		-0.007
		(-3.59)		(-3.57)		(-3.57)		(-3.55)
$Dispersion_t$		0.598		0.667		0.551		0.657
		(1.07)		(1.06)		(1.05)		(1.04)
$R^2(\%)$ *mean*	2.380	57.732	2.567	58.047	4.452	58.148	5.492	58.652
Median	1.521	59.582	1.472	60.562	1.411	59.989	1.658	60.883

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